Decentralized decision making under uncertainty

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Yale University

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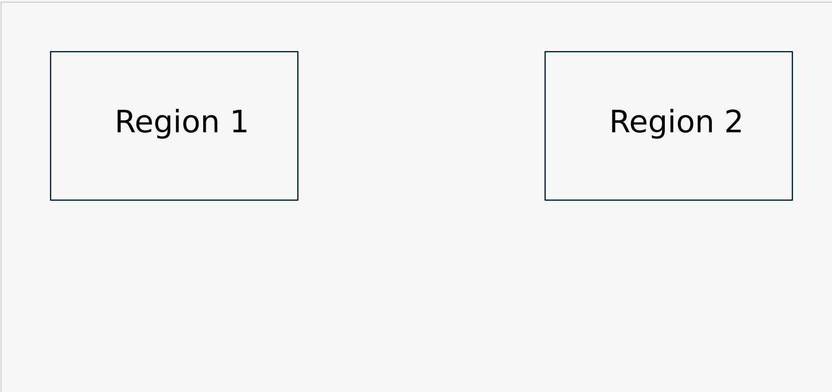
Decentralized systems
are ubiquitous

Interconnected power systems

Interconnected power systems



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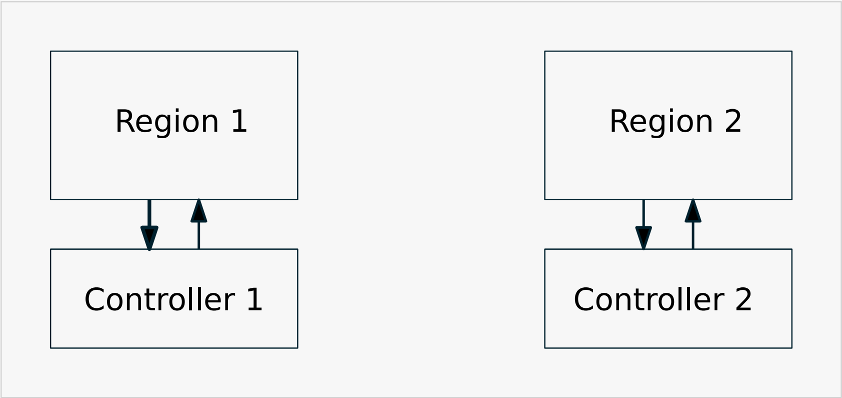

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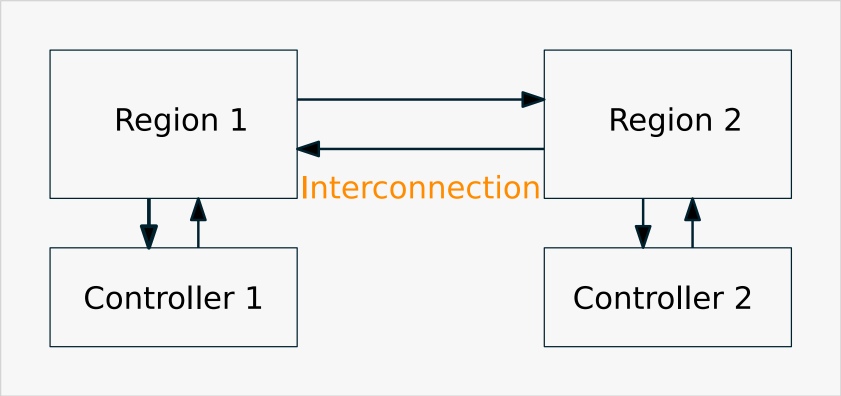

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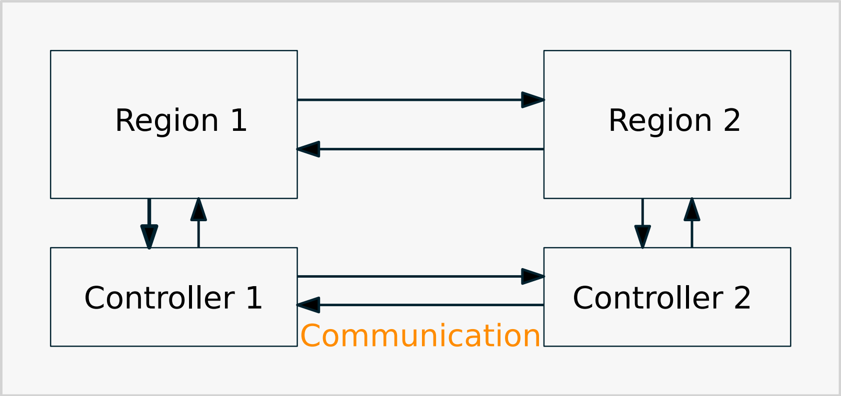

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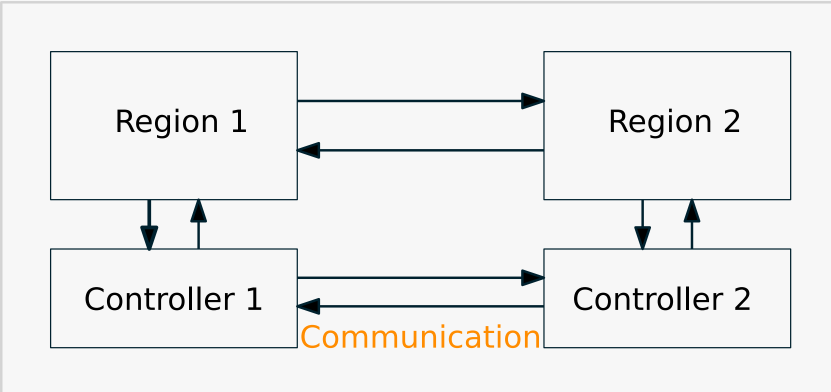
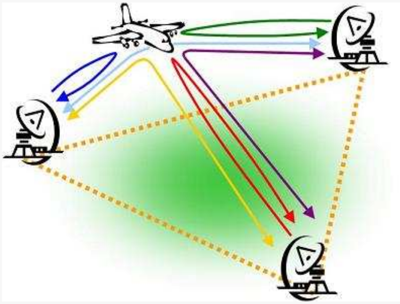


- Challenges
  - How to coordinate
  - What to communicate
  - How to communicate
  - When to communicate

Sensor and Surveillance networks



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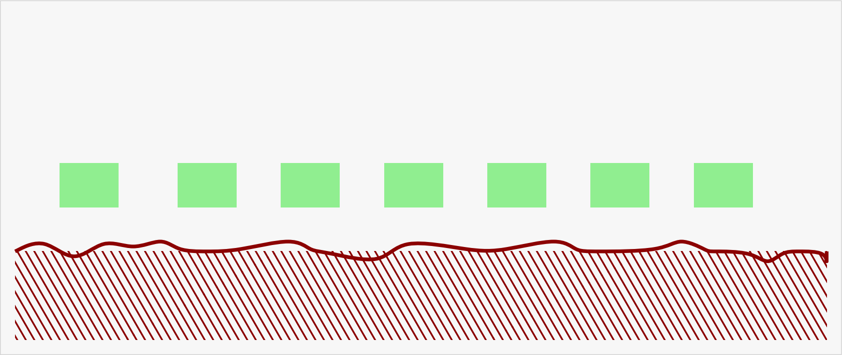

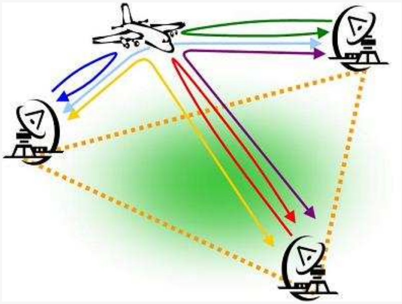

- Limited resources

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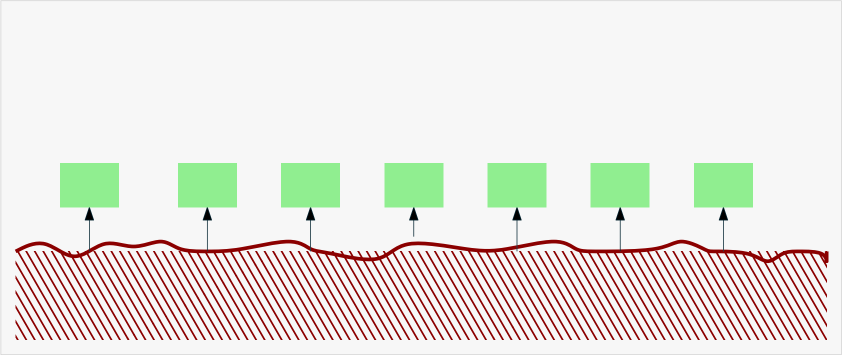

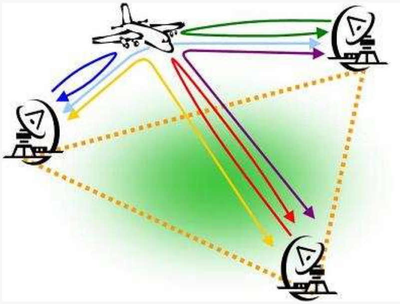

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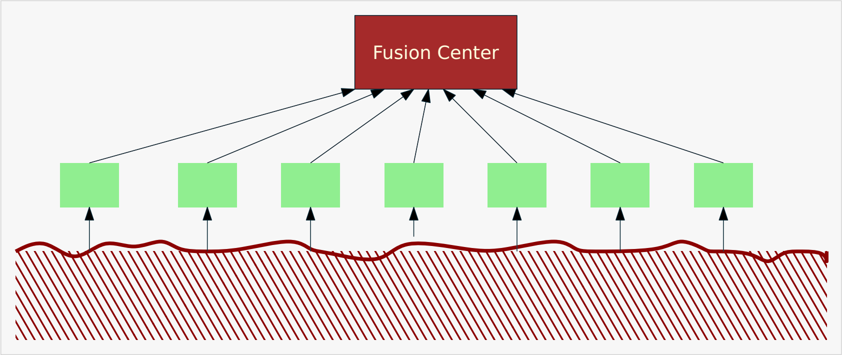

- Limited resources
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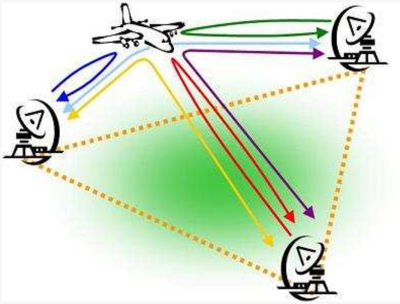
- Communication



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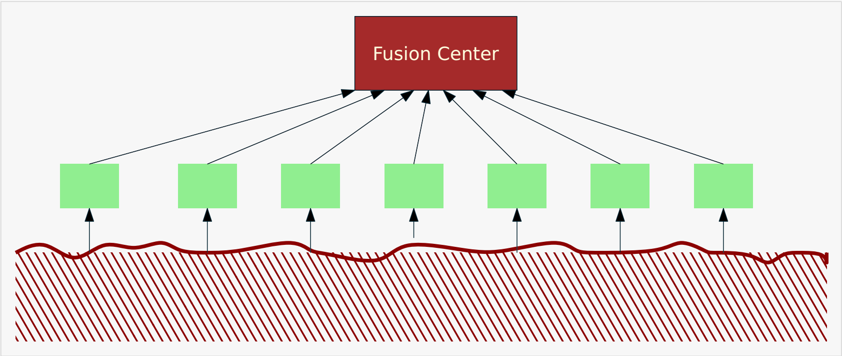

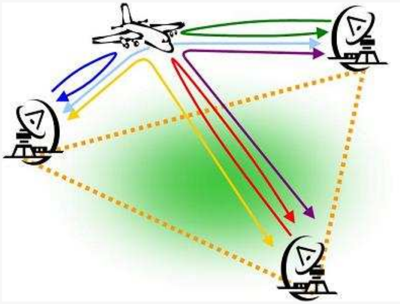

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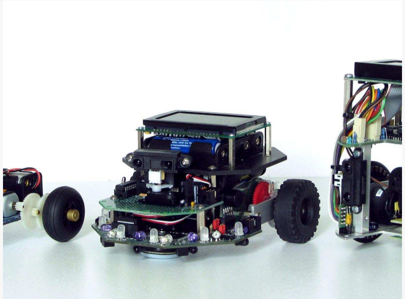
Challenges

- Real-time communication
- Scheduling measurements and communication
- Detect node failures

Networked control systems



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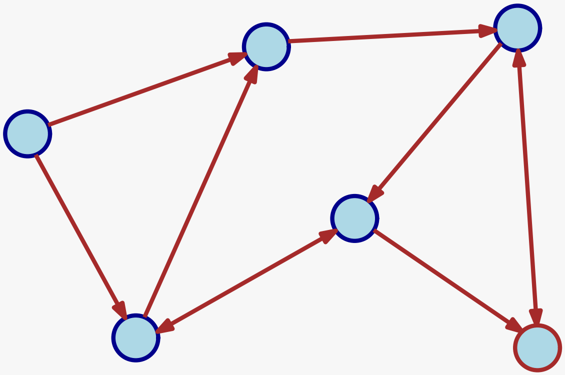

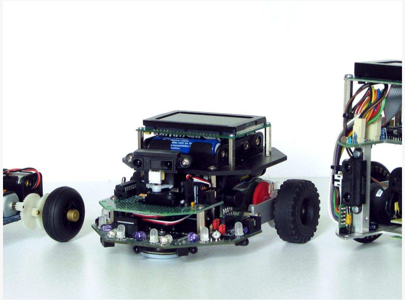

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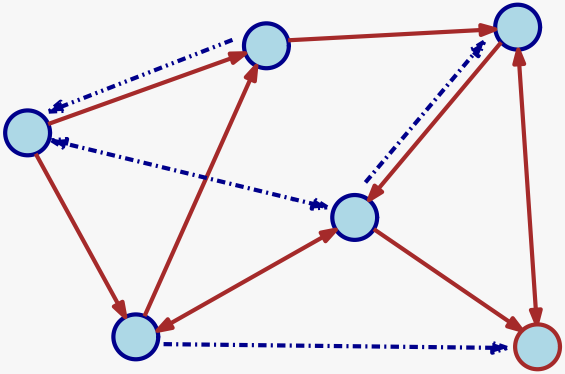

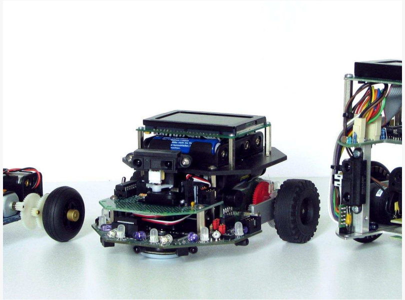

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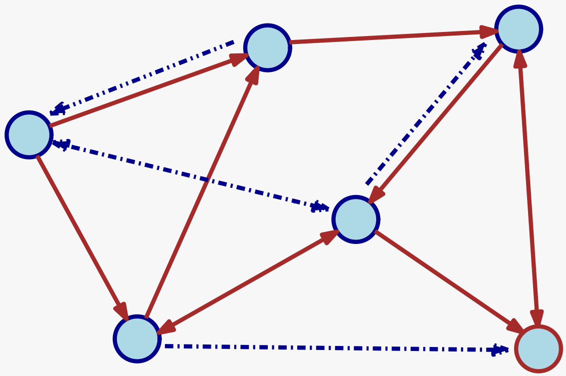

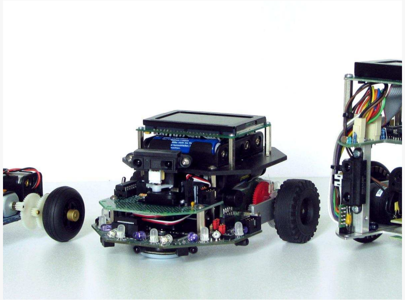

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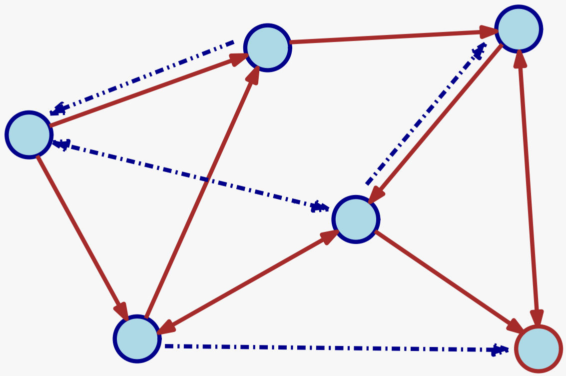

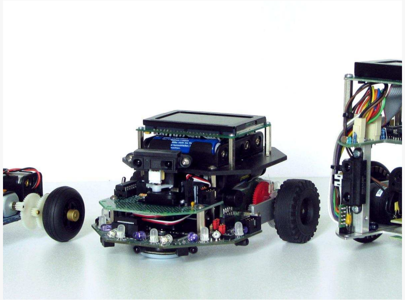

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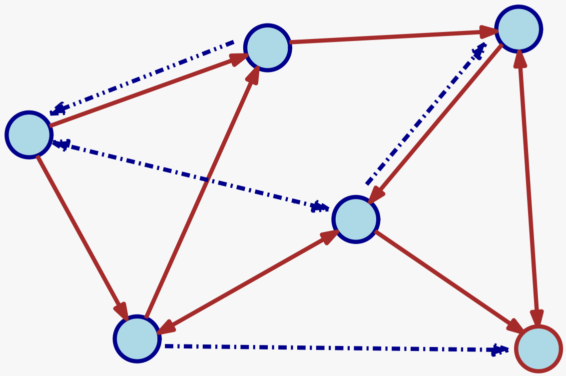

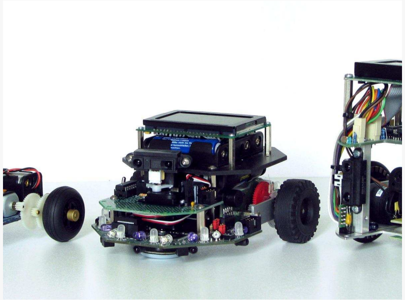

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  Decision making by multiple agents in stochastic dynamic environment

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- Exploiting domain knowledge

  Application specific modeling assumptions

Research Directions

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Delay sensitive communication

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- Optimal control over noisy channels

  Communication and coordination

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Research Directions

- **Real-time communication**

  Communication constraint

- **Optimal control over noisy channels**

  Coordination

- **Delayed sharing patterns**

  Coordination

- Communication over unknown channels

  Robust communication

- Calibration and validation of remote sensing observations

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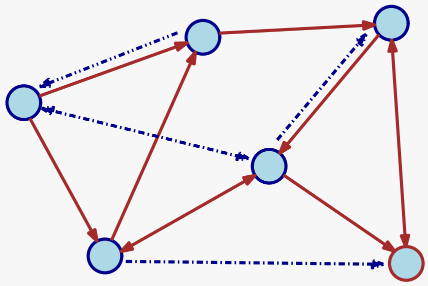
Real-time communication

Real-time communication

Simplest setup: A node observes a stream of data and has to communicate it to another node (over possibly noisy channels) within a fixed finite delay

- Integral component of many applications
  - Sensor and surveillance networks
  - Transportation networks
  - Fault diagnosis in power systems
  - Networked control systems



Importance and Challenges: An example

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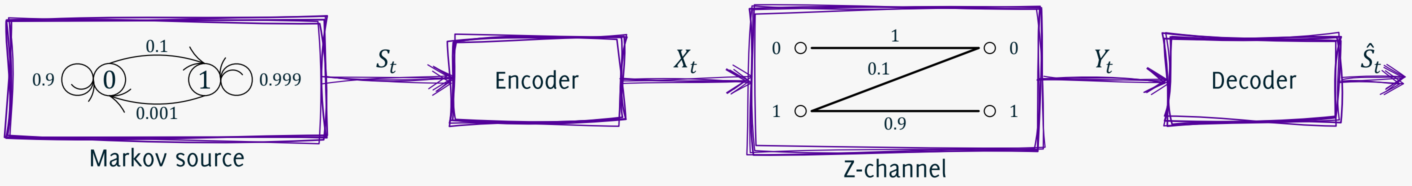

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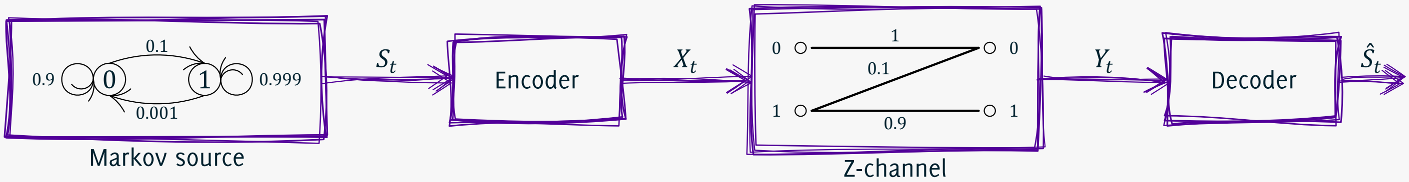

- Probability of error

$$\mathbb{P}(s_1 \neq \hat{s}_1) + \mathbb{P}(s_2 \neq \hat{s}_2) + \mathbb{P}(s_3 \neq \hat{s}_3)$$

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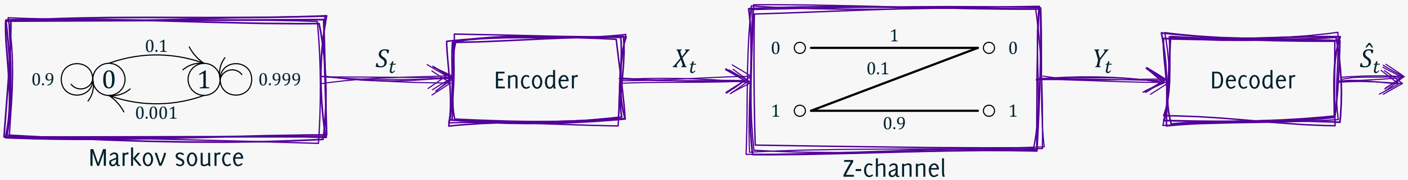

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$$x_1 = e_1(s_1), \quad x_2 = e_2(s_2), \quad x_3 = e_3(s_3)$$

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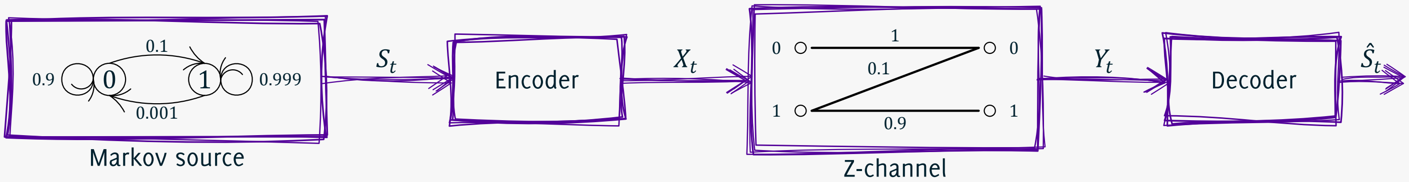

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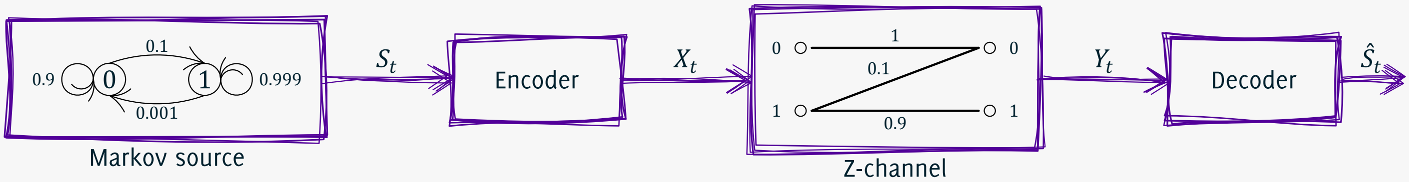

	Memoryless	Causal real-time	
Prob of error	0.1346	0.0564	240% better

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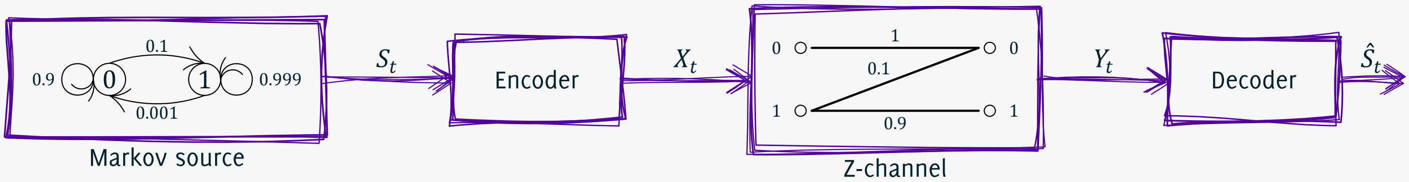

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	$O(10^3)$	$O(10^8)$	

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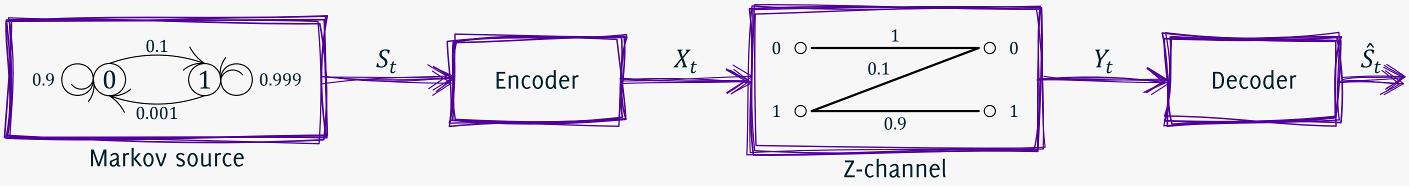

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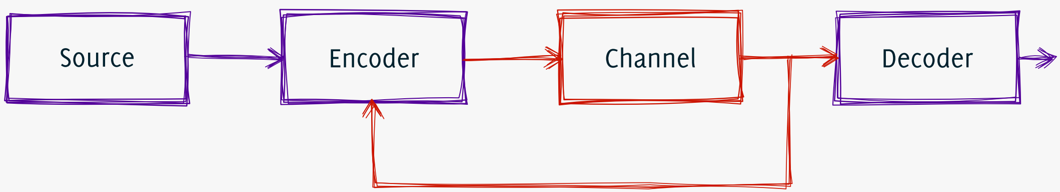
Can we search for optimal real-time strategies efficiently?

Literature Overview

- Encoder knows what decoder knows

Noiseless channel: Lloyd, 1977; Witsenhausen, 1979; Neuhoff and Gilbert, 1982; Linder and Lugosi, 2001; Weissman and Merhav, 2002; Linder and Zamir, 2006.



Noisy channel with noiseless feedback:
Walrand and Varaiya, 1982

Literature Overview

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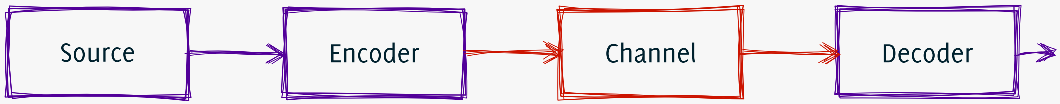

Finite memory: Gaarder and Slepian, 1982;
Mahajan and Teneketzis, 2006

Noisy channel: Teneketzis, 2006,
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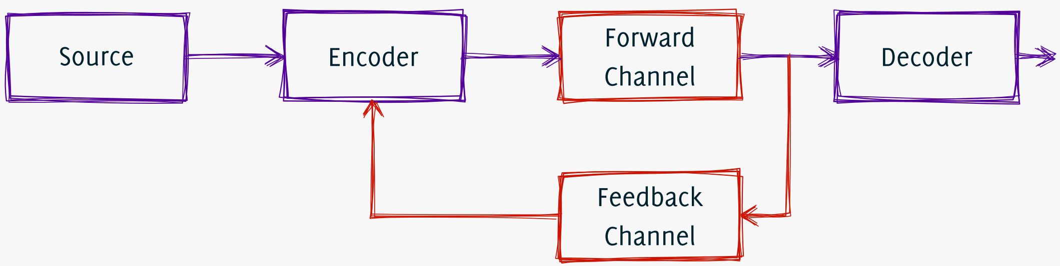

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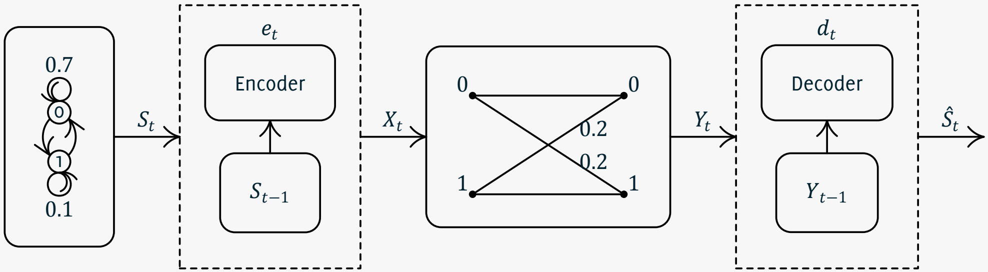

Finite Memory

Encoder

$$x_t = e_t(s_t, s_{t-1})$$

Decoder

$$\hat{s}_t = d_t(y_t, y_{t-1})$$

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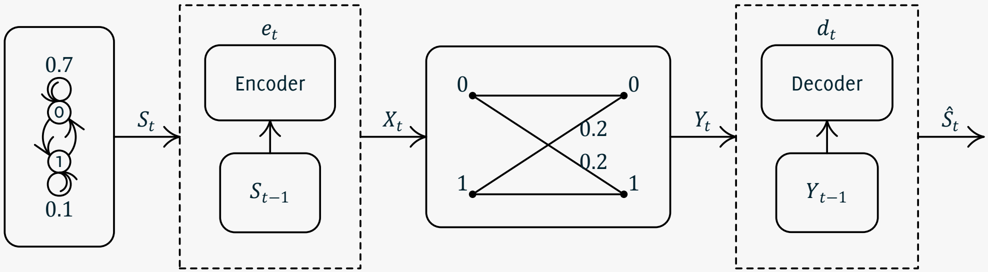

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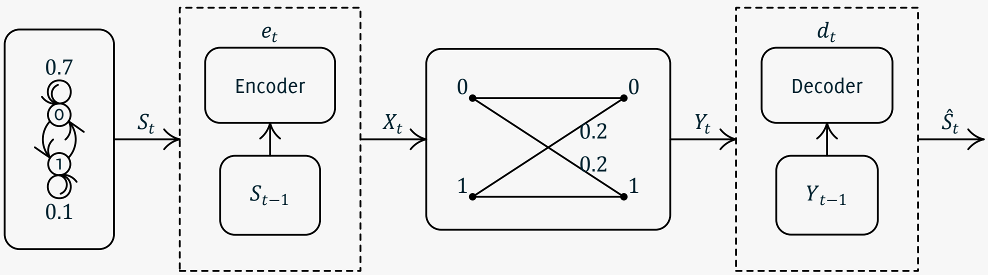

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- Performance

$$\mathcal{J}(E, D) = \lim_{T \to \infty} \mathbb{E}\left\{ \sum_{t=2}^{T} \beta^{t-1} \mathbb{P}(\hat{s}_t \neq s_{t-1}) \right\}$$

Gaarder and Slepian's approach

[Gaarder and Slepian, 1982]

- Choose a time invariant strategy $E = (e, e, \dots, e)$, $D = (d, d, \dots, d)$.

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- Repeat for all time invariant strategies.

so we deem it best to publish now the results we do have; albeit, incomplete and unsatisfactory as they are. Perhaps others will pick up the fallen torch and run more deftly!

Gaarder and Slepian, 1982

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- Steady-state distribution of a Markov chain is discontinuous in its transition matrix

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- Steady-state distribution of a Markov chain is discontinuous in its transition matrix
- For some (E, D), the Markov chain may not have a unique steady-state distribution
  - Multiple recurrence classes $\Rightarrow$ uncountable steady-state distributions

Determining optimal encoders and decoders

None of the existing approaches work

Difficulty with other approaches

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- Brute force search is computationally challenging

  Number of communication strategies: $\left(|\mathcal{X}|^{|\mathcal{S}|^2} |\hat{\mathcal{S}}|^{|\mathcal{Y}|^2} \right)^T$

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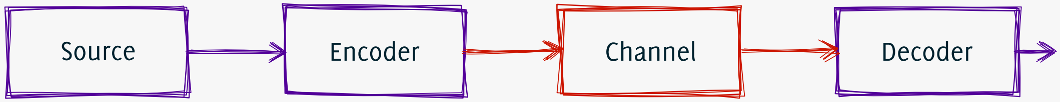

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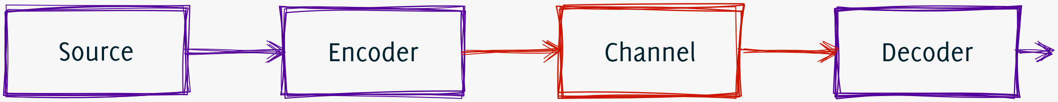

- Orthogonal search (Coordinate descent)
  - May not converge
  - Gives only local optima

Global Optimization

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- Common knowledge
  - What can two agents with different information agree upon?
  - Key notion in finding information states

Research Contributions

- Sequential decomposition
  - Sequential search algorithm
  - Exponentially reduces the search complexity
- Information state
  - No existing methodology
  - Provided an axiomatic approach
- Common knowledge
  - What can two agents with different information agree upon?
  - Key notion in finding information states
- Finite or infinite time-steps
  - No priori approach for infinite time-steps
  - Proposed approach works for both

Sequential decomposition

- Divide and Conquer

  Algorithm: One step optimization → sequence of nested optimizations

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Sequential decomposition

- Divide and Conquer

Exponential reduction in
the search complexity

$$O\left((2^A)^T \right) \to O(T \cdot K \cdot 2^A)$$

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Sequential Decomposition

First example of sequential decomposition for
optimal solution of
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- How do we choose information state π_t

No previous known technique for finding
information states for decentralized systems

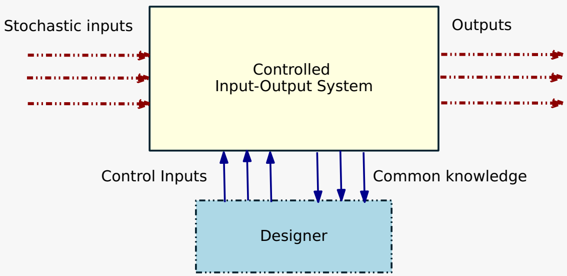
An axiomatic approach to choosing information state

Choosing information state

[Mahajan, 2008, Mahajan and Teneketzis 2008, 2009b]

Do not think in terms of encoders or decoders;

think in terms of a system designer



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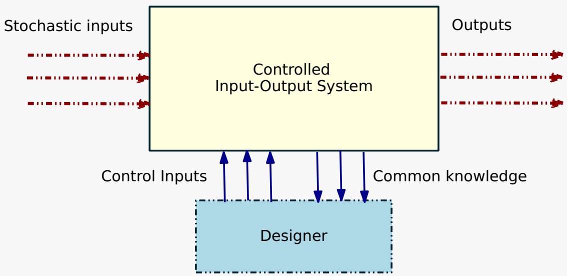

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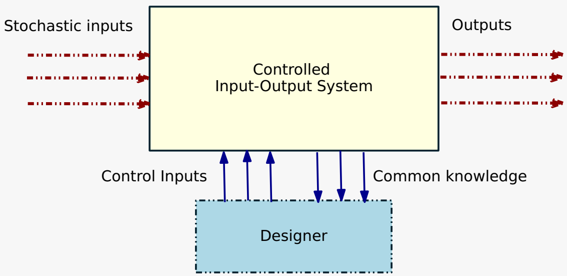

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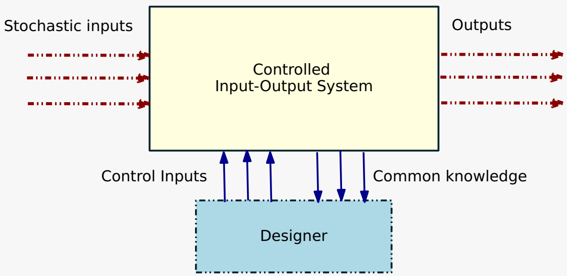

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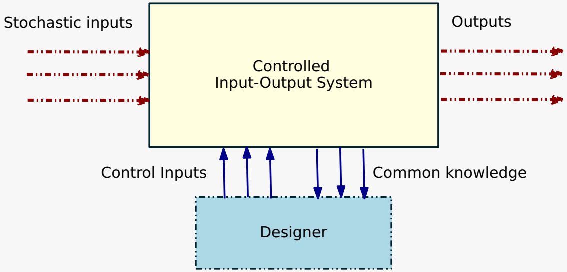

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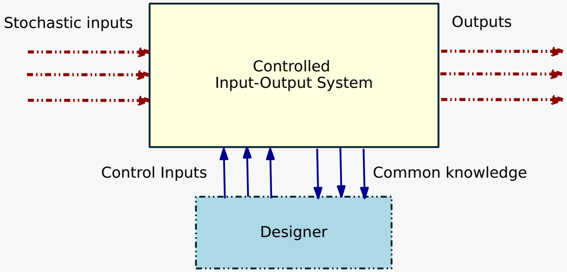

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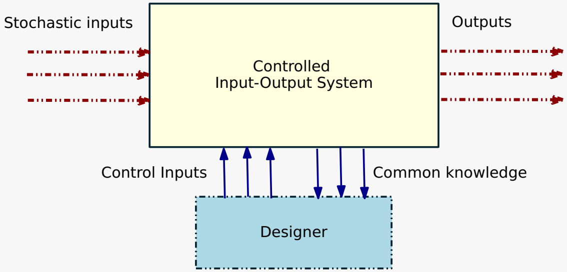

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$\mathbb{P}$(State for i/o mapping | common knowledge)

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$$\pi_t = \mathbb{P}(s_t, s_{t-1}, y_{t-1} \mid e_{1:t-1}, d_{1:t-1})$$

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Information state Seq decomposition

[Mahajan and Teneketzis, 2009b]

Finite horizon: An optimal communication strategy can be determined by the solution of the following nested optimality equations

$$V_T(\pi_T) = \min_{e_T, d_T} \mathbb{E}\Big[\, \mathbb{P}(\hat{s}_T \neq s_{T-1}) \,\Big|\, \pi_T, e_T, d_T \,\Big]$$

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Infinite horizon: . . . fixed point equation

$$V(\pi) = \min_{e,d} \mathbb{E}\Big[\, \mathbb{P}(\hat{s}_t \neq s_{t-1}) + \beta V(\pi_+) \,\Big|\, \pi, e, d \,\Big]$$

Optimal communication scheme

- Example with $\beta = 0.9$

$$(e_t, d_t) = \begin{pmatrix} s_t & , & 0 \\ s_{t-1} \oplus s_t & , & y_{t-1} \oplus y_t \\ s_{t-1} & , & y_{t-1} \oplus y_t \end{pmatrix}_{t \pmod 3}$$

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Source	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Encoder							
Decoder							
Estimate	$-$	s_1	s_2	s_3	s_4	s_5	s_6

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This restriction made their setup extremely difficult to solve.

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Design of decentralized systems requires a paradigm shift

Summary

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- Qualitative difference in results

Even for infinite horizon problems,
time-invariant strategies may not be optimal

Generality of the approach

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- General model for real-time communication
  - Full-memory encoder or decoder (Mahajan and Teneketzis, 2009b)
  - Channels with memory (Mahajan and Teneketzis, 2009b)
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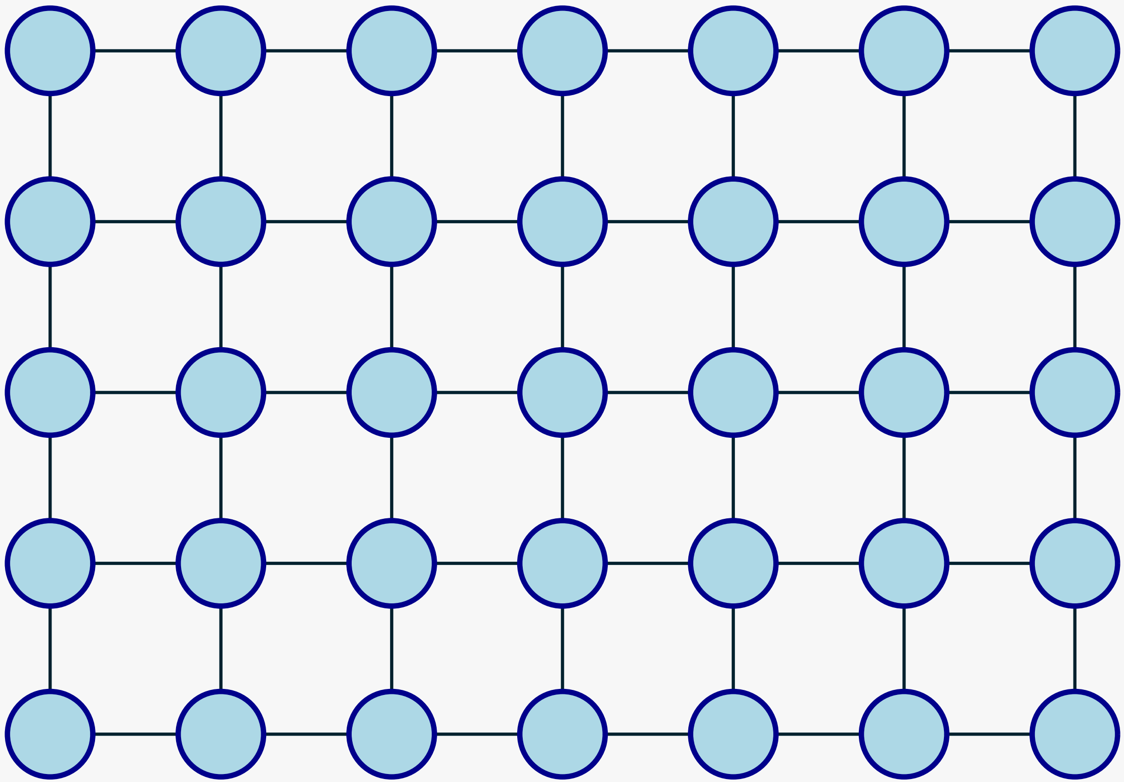
- Delayed sharing patterns

  A 40 year old open problem

Delayed Sharing Patterns



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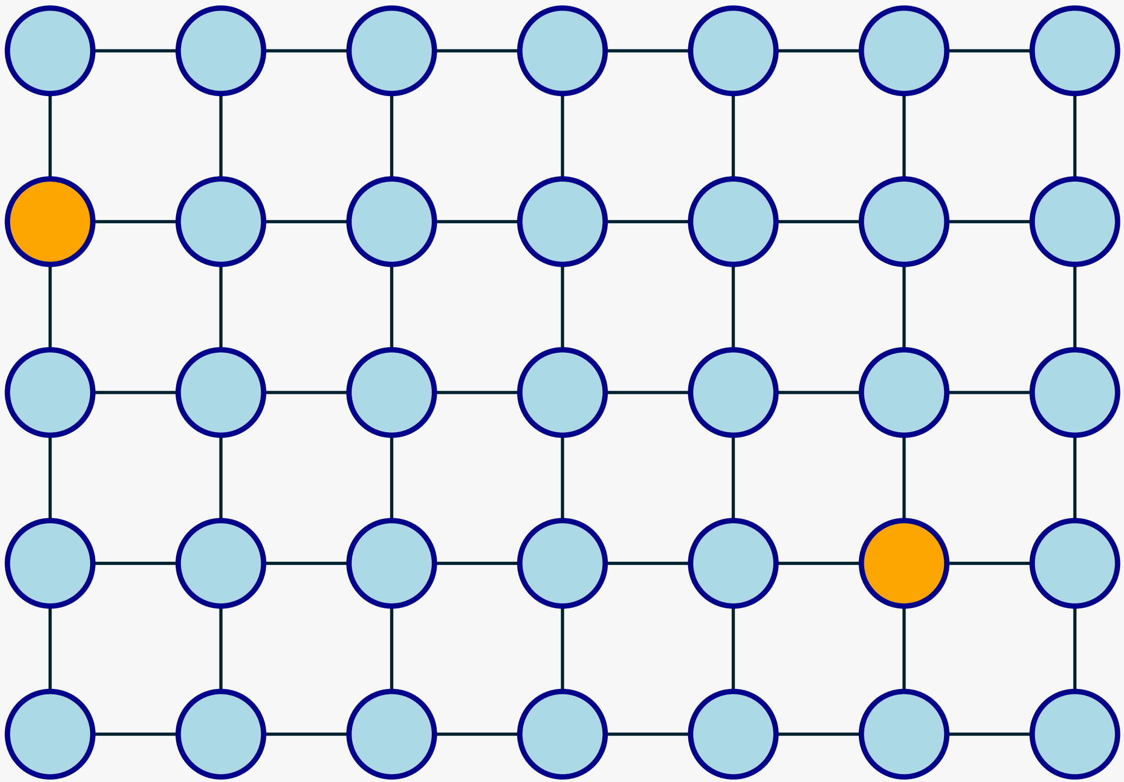

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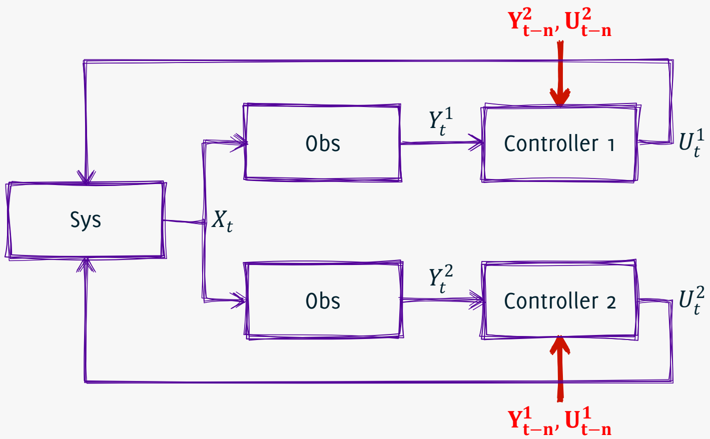

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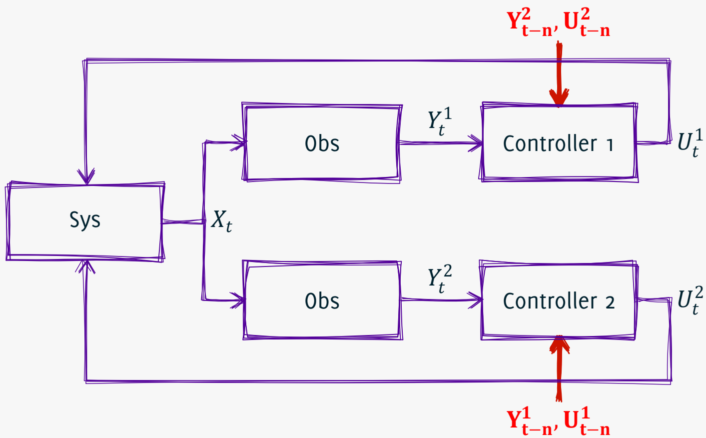

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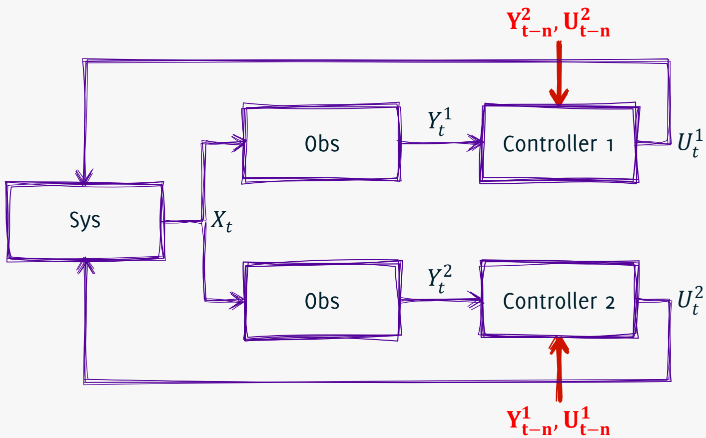

$$U_t^i = g_t^i(C_t, L_t^i)$$

where

$$C_t = \left(\begin{bmatrix} Y_1^1 \\ \vdots \\ Y_{t-n}^1 \end{bmatrix}, \begin{bmatrix} Y_1^2 \\ \vdots \\ Y_{t-n}^2 \end{bmatrix}, \begin{bmatrix} U_1^1 \\ \vdots \\ U_{t-n}^1 \end{bmatrix}, \begin{bmatrix} U_1^2 \\ \vdots \\ U_{t-n}^2 \end{bmatrix} \right) \qquad L_t^i = \left(\begin{bmatrix} Y_{t-n+1}^1 \\ \vdots \\ Y_t^1 \end{bmatrix}, \begin{bmatrix} U_{t-n+1}^1 \\ \vdots \\ U_{t-1}^1 \end{bmatrix} \right)$$

Common Information

Local Information

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  Without loss of optimality, each controller can replace C_t by $\mathbb{P}(X_{t-n} \mid C_t)$

- Varaiya and Walrand (1979)
  - True for $n = 1$
  - False for $n > 1$

Open problem for 40 years

Does a information state for C_t exist?

How do we find such a information state?

Importance of the problem

- Applications (of one step delay sharing)
  - Power systems: Altman *et. al*, 2009
  - Queueing theory: Kuri and Kumar, 1995
  - Communication networks: Grizzle *et. al*, 1982
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- Conceptual significance
  - Understanding the design of networked control systems
  - Bridge between centralized and decentralized systems
  - Insights for the design of general decentralized systems

Solution approach

[Nayyar, Mahajan, and Teneketzis, 2010]

- Common knowledge between all agents

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$$\begin{aligned} \pi_t &= \mathbb{P}(\text{state for i/o mapping} \mid \text{common knowledge}) \\ &= \mathbb{P}(X_t, L^1_t, L^2_t \mid C_t, g^1_{1:t-1}, g^2_{1:t-1}) \end{aligned}$$

Solution approach

[Nayyar, Mahajan, and Teneketzis, 2010]

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Structure of optimal control law

Without loss of optimality, each controller can replace C_t by $\mathbb{P}(X_t, L^1_t, L^2_t \mid C_t, g^1_{1:t-1}, g^2_{1:t-1})$

Can also write a sequential decomposition based on this information state

A systematic approach can easily resolve long-standing conceptual difficulties in decentralized systems

Conclusions
Optimal design of decentralized systems

Decentralized system: Salient features

- Multiple agents

  Decision making by multiple agents in stochastic dynamic environment

- Coordination issues

  All agents must coordinate to achieve a system-wide objective

- Communication constraints

  Data must be communicated within fixed finite delay

- Robustness

  System model may not be known completely

- Exploiting domain knowledge

  Application specific modeling assumptions

Decentralized systems: Research directions

- Real-time communication

  Delay sensitive communication

- Optimal control over noisy channels

  Communication and coordination

- Delay sharing pattterns

  Coordination

- Communication over unknown channels

  Robustness

- Calibration and validation of remote sensing observations

  Exploiting domain knowledge

Summary

- Systematic approach to design decentralized systems

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- Algorithm to sequentially synthesize optimal controllers

  Based on information states

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  - Find common knowledge
  - Find state for i/o mapping
  - $\mathbb{P}$(state for i/o mapping | common knowledge)

Summary

- Systematic approach to design decentralized systems
- Algorithm to sequentially synthesize optimal controllers

  Based on information states

- Axiomatic approach to find information states
  - Find common knowledge
  - Find state for i/o mapping
  - $\mathbb{P}$(state for i/o mapping | common knowledge)
- Delayed sharing pattern

  Able to resolve a long standing open conjecture

Future Directions

- Control of power systems
  - Renewable energy: unpredictable generation
  - Energy markets: Game theoretic considerations


Future Directions

- Control of power systems


  - Renewable energy: unpredictable generation
  - Energy markets: Game theoretic considerations

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  - Climate change: cheap yet reliable monitoring
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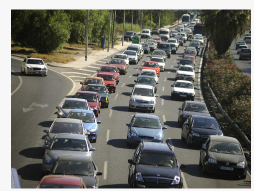
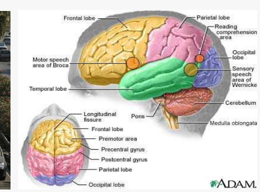
Future Directions

- **Control of power systems**
  - Renewable energy: unpredictable generation
  - Energy markets: Game theoretic considerations

- **Environmental sensor networks**
  - Climate change: cheap yet reliable monitoring
  - Calibration validation of remote sensing observations Time varying sampling

- **Control and coordination**
  - Transportation networks
  - Bioscience and medicine
  - . . .





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- A. Mahajan and D. Teneketzis, Optimal performance of networked control systems with non-classical information structures, *SIAM Journal of Control and Optimization*, 2009
- A. Mahajan and D. Teneketzis, Optimal performance of feedback control systems with limited communication over noisy channels, *45th IEEE Conference on Decision and Control (CDC)*, 2006

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- A. Nayyar, A. Mahajan and D. Teneketzis, Optimal control strategies om delayed sharing information structures, *IEEE Transactions on Automatic Control*, Feb 2010 (submitted)
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- A. Mahajan and S. Yüksel, Measure and cost dependent properties of information structures, *American Control Conference (ACC)*, 2010
- A. Mahajan and S. Tatikonda, Sequential team form and its simplification using graphical models, *47th Allerton Conference*, 2009
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Thank you

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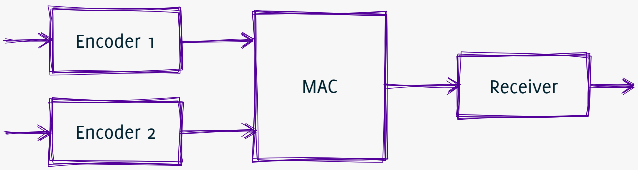
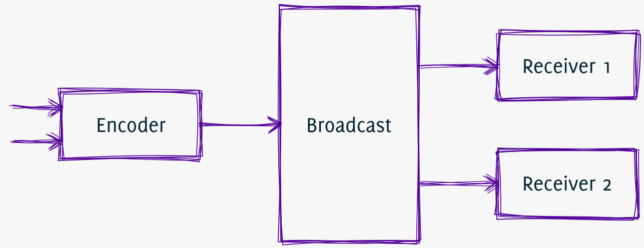
Towards a theory of real-time network communication

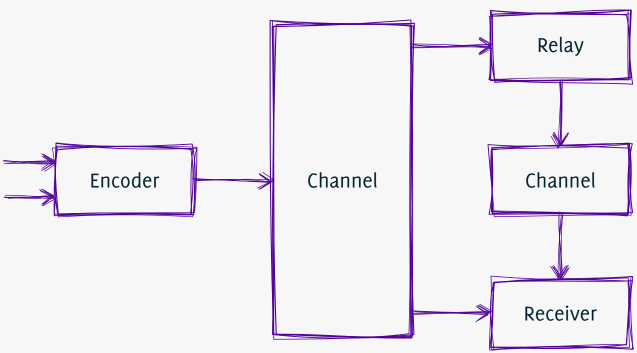
Towards a theory of real-time network communication



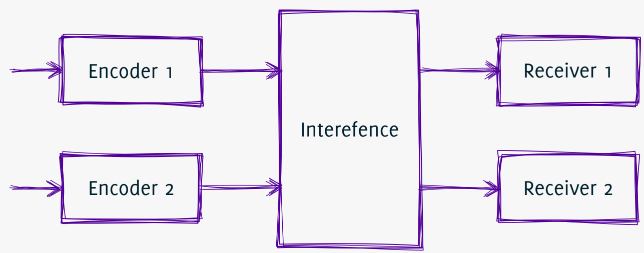
Multiple access channel (M, 2009a)



Broadcast channel (M, 2009b)



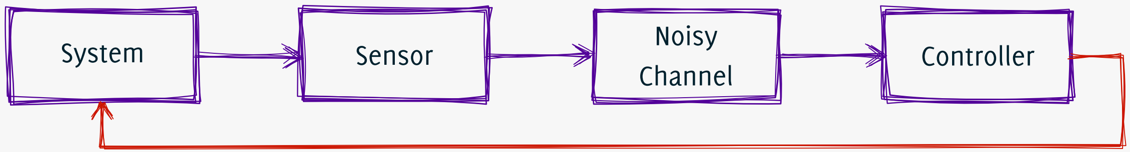
Relay channel



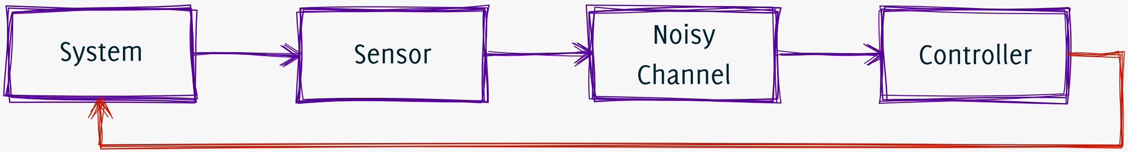
Interference channel

A surprisingly related problem

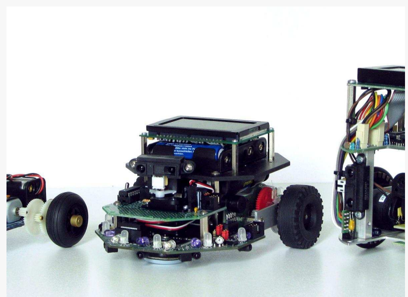
Optimal control over noisy channels

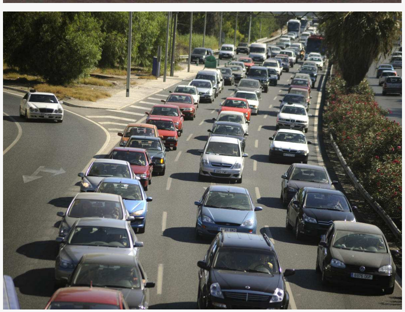
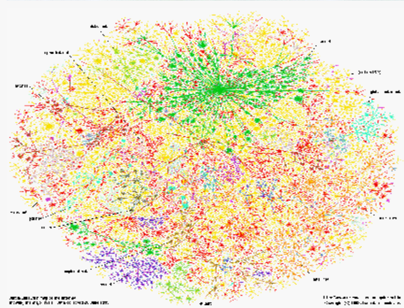
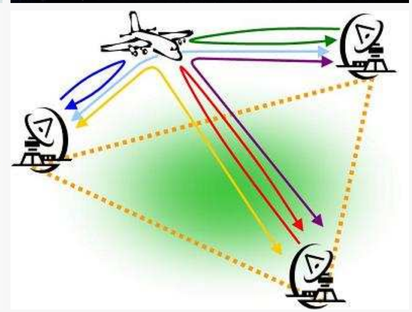


Optimal control over noisy channels



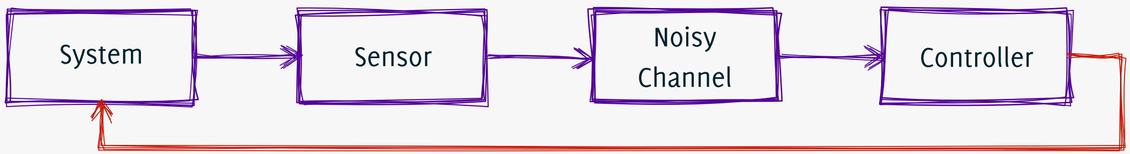
- Simplest example of optimal control of networked systems

Optimal control over noisy channels

[M and Teneketzis, 2009a]



- Salient features
  - Noisy channel
  - Performance optimization

Optimal control over noisy channels

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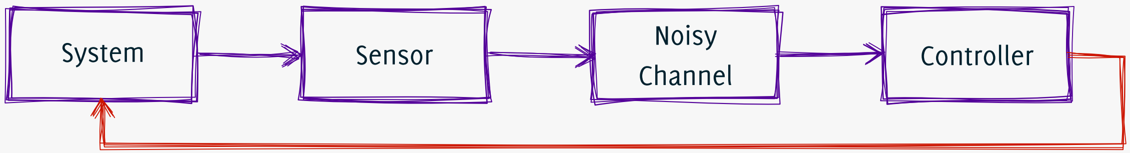

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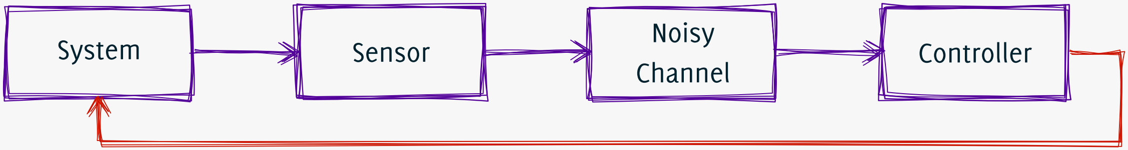

- Salient features
  - Noisy channel
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- Control vs (real-time) communication
  - **Similar design difficulties**

    non-comparable information

    - No brute force
    - No Markov decision theory
    - No orthogonal search

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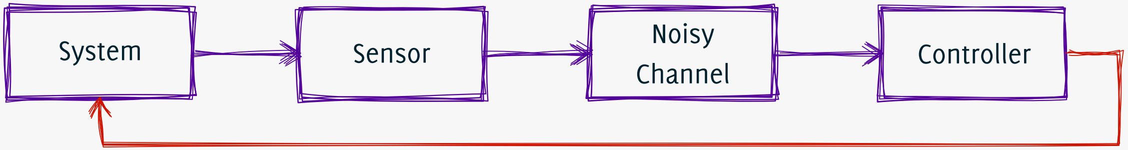

- Salient features
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- Control vs (real-time) communication
  - **Similar design difficulties**

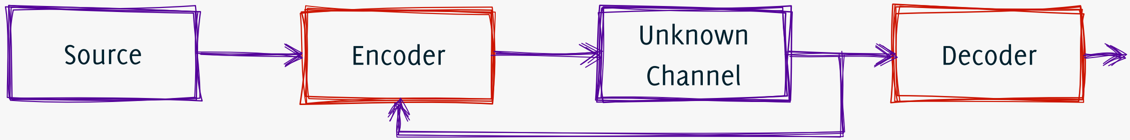
    non-comparable information

    - No brute force
    - No Markov decision theory
    - No orthogonal search
  - **Similar solution approach works**

    Algorithm to sequentially search for the optimal encoding and control strategies

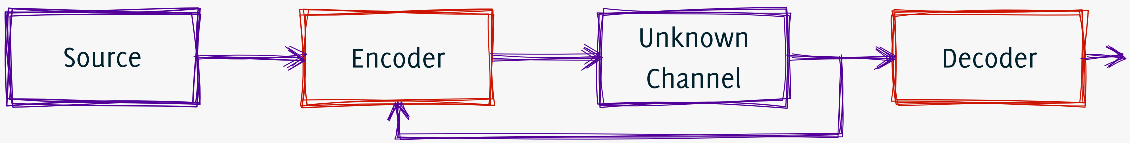
Feedback communication over unknown channels

Feedback communication over unknown channels



- Trade off between learning and communication

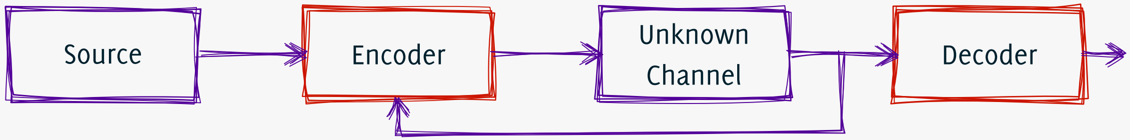
Feedback communication over unknown channels



- Trade off between learning and communication
- Performance criterion:

Error exponent $E = \lim_{n\to\infty} \mathbb{P}^{(n)}_{\text{error}}$

Feedback communication over unknown channels



- Trade off between learning and communication
- Performance criterion:

  Error exponent $E = \lim_{n\to\infty} \mathbb{P}^{(n)}_{\text{error}}$

- Training based scheme
  - Send training sequence
  - Estimate channel and communicate

Feedback communication over unknown channels

- Proposed coding scheme

$$E_{\text{proposed}} = \alpha E_{\text{known}}$$

Feedback communication over unknown channels

- Proposed coding scheme

$$E_{\text{proposed}} = \alpha E_{\text{known}}$$

- Main insights
  - Need to send multiple training sequences
  - Channel estimation for each training sequence should be done independently

Feedback communication over unknown channels

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- Reference

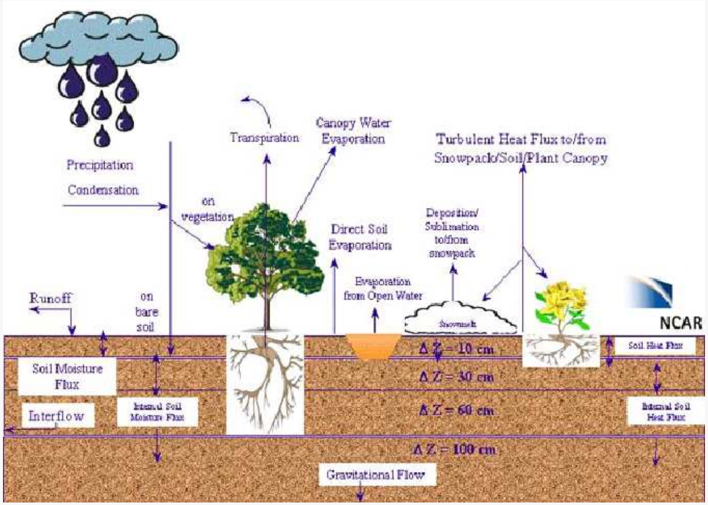
A. Mahajan and S. Tatikonda, Opportunistic capacity and error exponents of compound channel with feedback, *IEEE Trans of Info Theory*, 2010 (submitted)

Calibration and Validation of Remote sensing observations

Monitoring soil moisture

- Measurement need for earth science

  NASA Earth Science focus:
  climate, carbon, weather,
  water, surface, and atmosphere

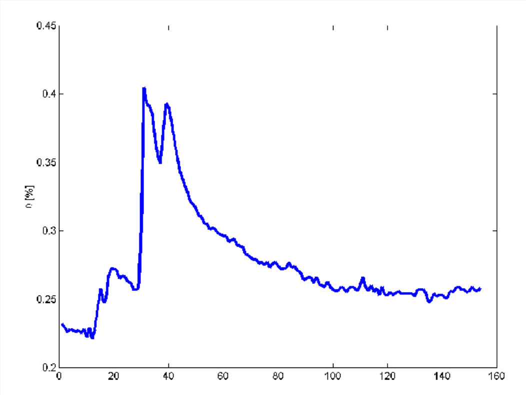


- Challenges

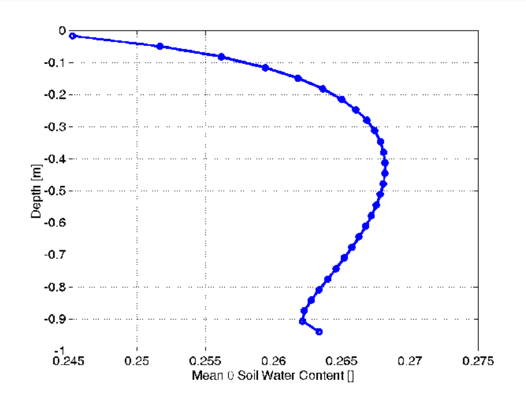
  - Complicated variation
    Depends on temperature, vegetation, precipitation, soil texture, topology, etc.

  - Remote sensing gives coarse estimates
    $O(1\text{km})$ to $O(10\text{km})$

Variation of Soil Moisture

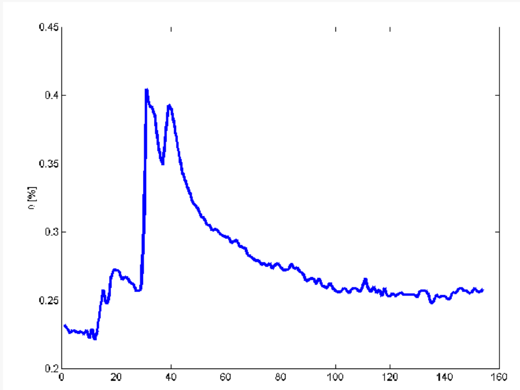


Variation with time



Variation with depth

Variation of Soil Moisture



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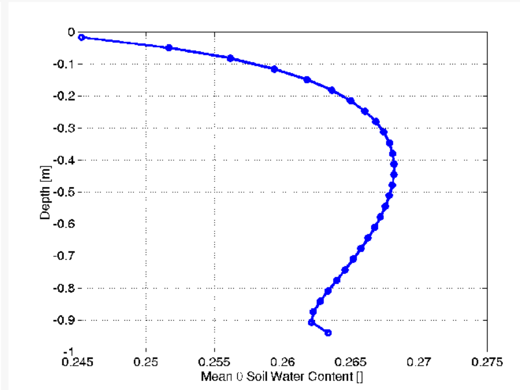
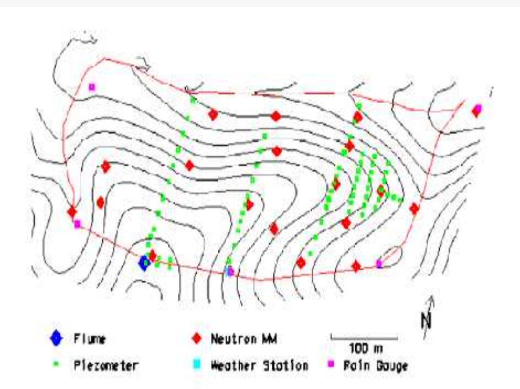
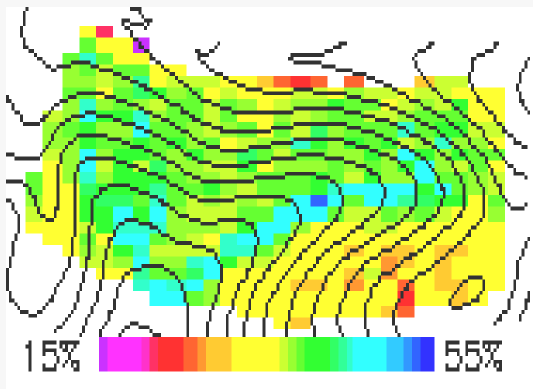


Variation with depth

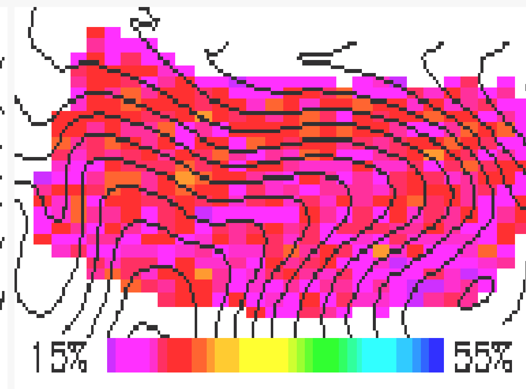


Topology



After rainfall

Follows topology



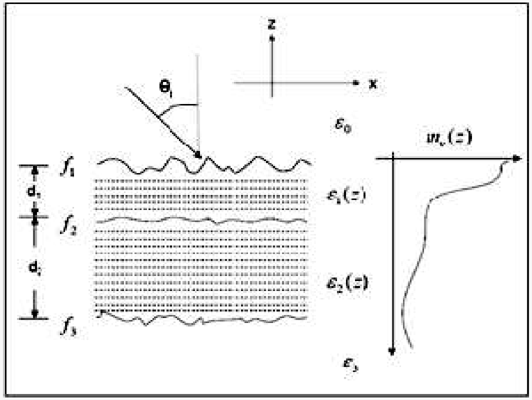
After dry run

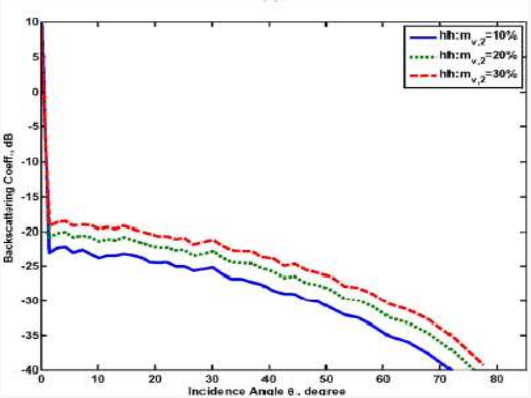
Follows vegetation

Radar measurements

- Backscattering coefficient

  Depends on incidence angle, soil depth, soil type, and vegetation cover





Sensor Scheduling

- Limited battery life

  Measurement and communication consumes energy

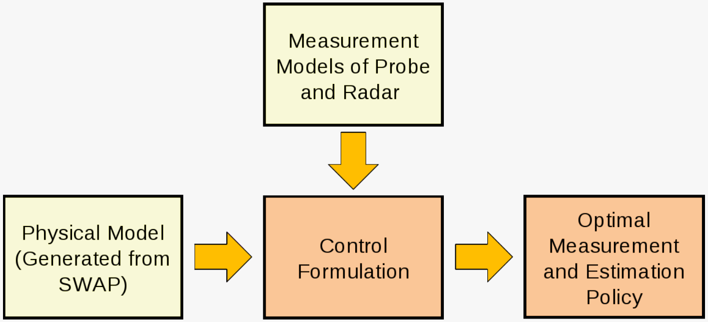
- Sleep scheduling

  Does the sensor need to switch on its radio to determine when to take a measurement?

- Sensor placement

  Need to cover a large area to match satellite footprint

Overview

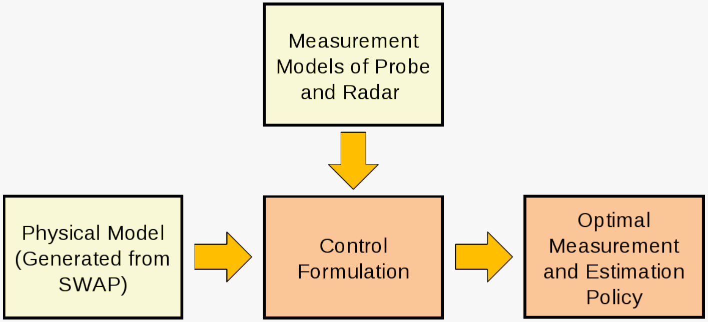


Measurement Models of Probe and Radar
Physical Model (Generated from SWAP)
Control Formulation
Optimal Measurement and Estimation Policy

Overview

- Physical model

  Use a community standard numerical model developed over 35 years (SWAP)



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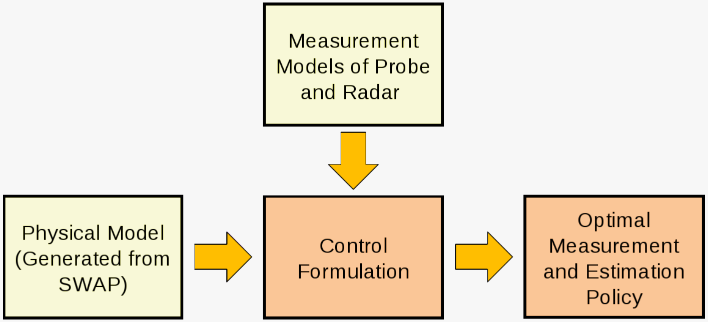
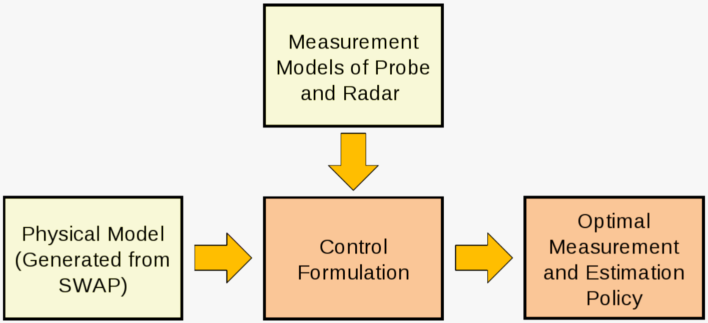


Measurement model

- Forward model for electromagnetic backscattering
- Sensor observation model based on calibration curves

Overview



- **Physical model**

  Use a community standard numerical model developed over 35 years (SWAP)

- **Measurement model**

  - Forward model for electromagnetic backscattering
  - Sensor observation model based on calibration curves

- **Optimal control formulation**

  Use physical and measurement model to guide sensor scheduling and measurement

Overview

- Physical model

Obtain a scalable algorithm
for sensor scheduling

  - Sensor observation model based on calibration curves

- Optimal control formulation

  Use physical and measurement model to guide sensor scheduling and measurement

Field Testing

- Location

  Matthaei Botanical Gardens, Ann Arbor, MI

- Multiple nodes at 3 depths

http://soilscape.eecs.umich.edu

References: Soil moisture measurement

- M. Moghaddam, D. Entekhabi, Y. Goykhman, K. Li, M. Liu, A. Mahajan, A. Nayyar, D. Shuman, and D. Teneketzis, A wireless soil moisture smart sensor web using physics-based optimal control: concept and initial demonstrations, *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 2010 (accepted)
- D. Shuman, A. Nayyar, A. Mahajan, Y. Goykhman, K. Li, M. Liu, D. Teneketzis, M. Moghaddam, D. Entekhabi, Measurement scheduling for soil moisture sensing: From physical models to optimal control, *Proceedings of the IEEE*, (in revision)
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  - Communication over unknown channels
  - Calibration and validation of remote sensing observations