Static Team with Common Information

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IFAC 2017 World Congress

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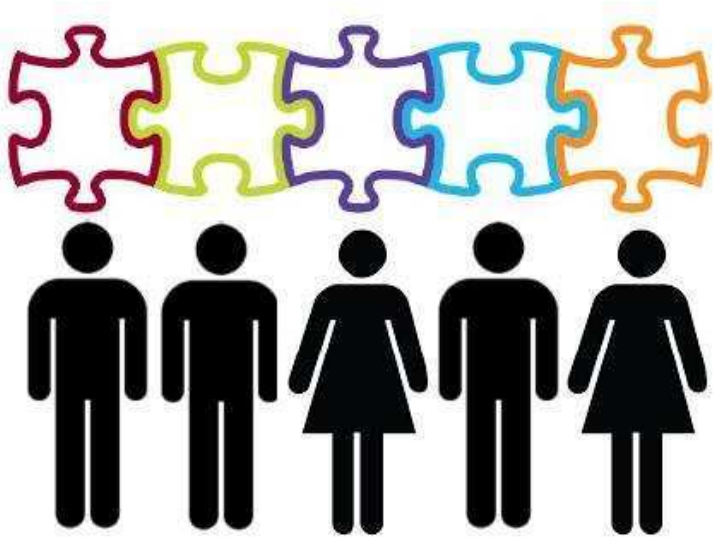
July 13, 2017

Introduction

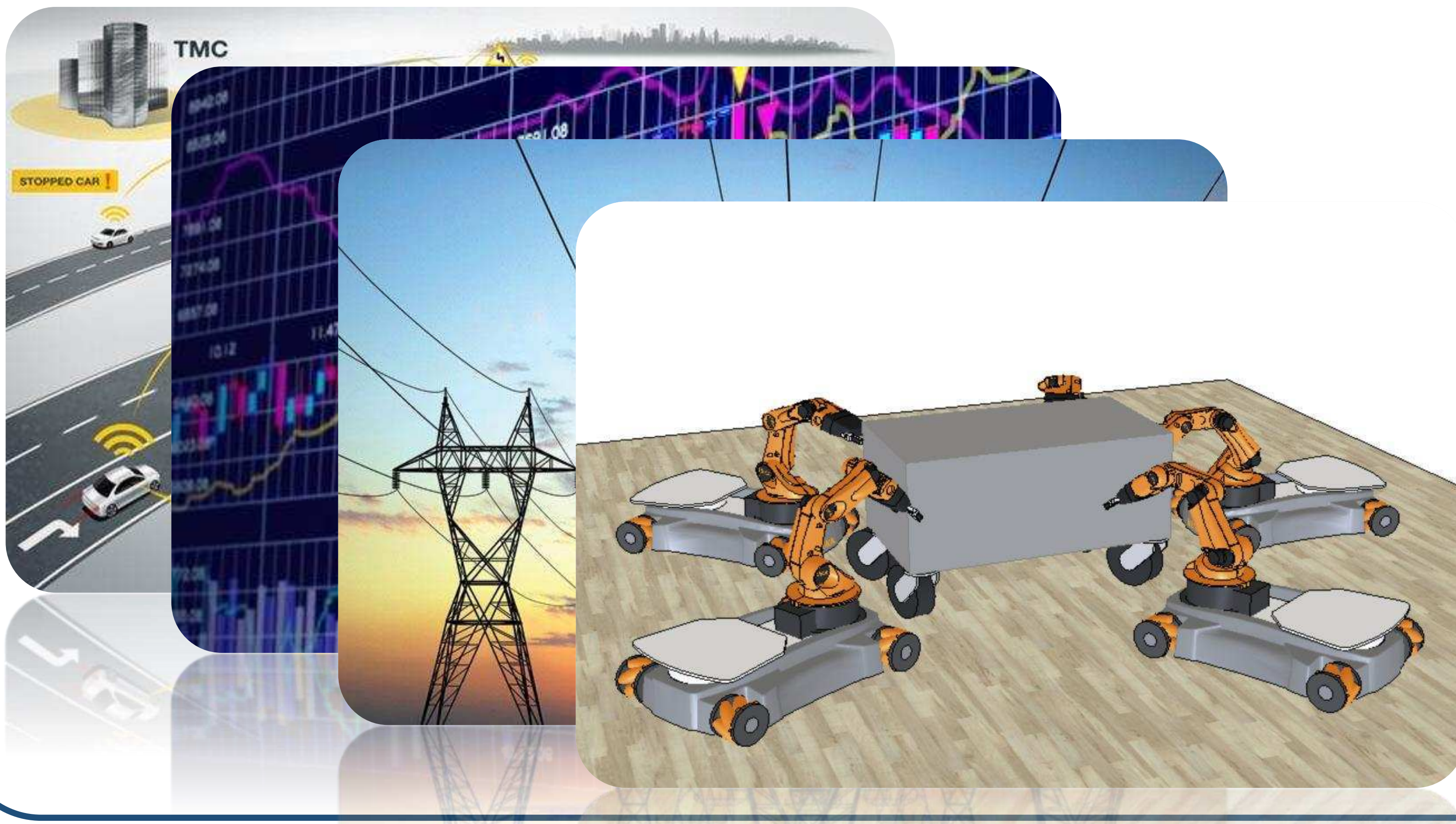
Decision Making:

- Centralized
- Decentralized
  - Game Problem: Individual objectives
  - Team Problem: Single common objective



Applications

- Traffic Management Systems
- Economics
- Interconnected Power Systems
- Multi robot control systems



Classification of Teams

Teams

- Static team: Actions of agents do not affect observations of others.
- Dynamic team: Actions of agents affect observations of others.

Solution Concepts

- Globally optimal solutions: Optimal solution among all possible solutions.
- Equilibrium solutions: Unilateral deviation by any player does not improve its performance.

Motivation

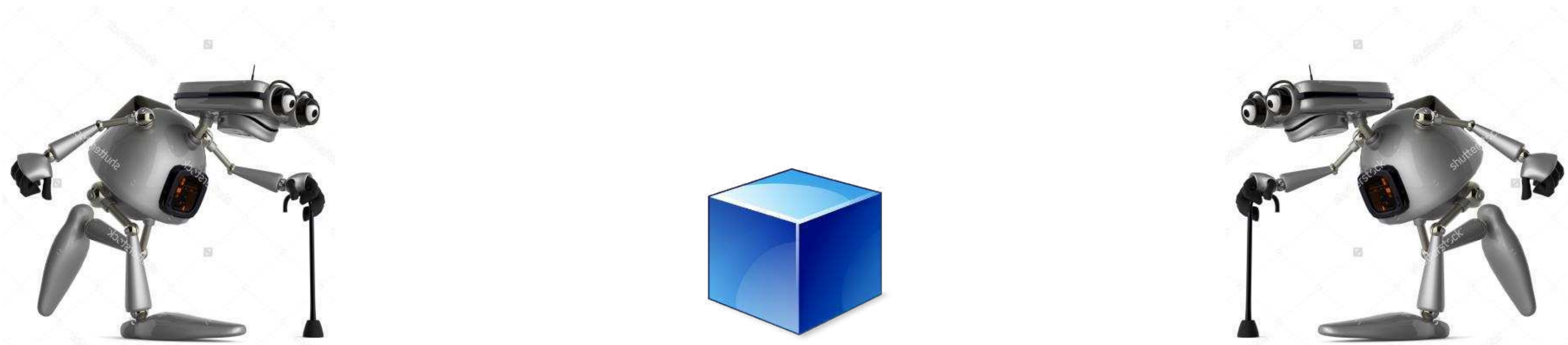
- In a team problem what is the performance improvement if we can add a sensor that broadcasts its observation to all agents?
- It's worthwhile only if this improvement in performance is greater than cost of the sensor.

We call the improvement the value of information.

In this paper we evaluate the value of common information in static LQG teams.

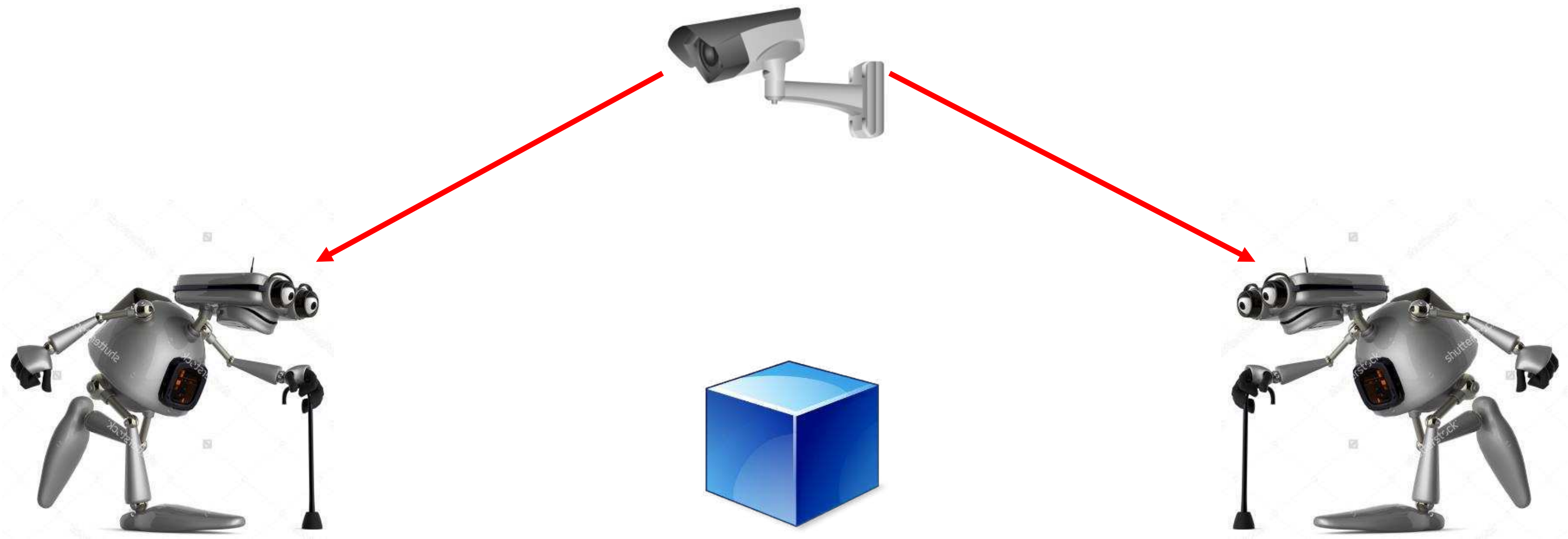
Example

- 2 robots with an object.
- Centralized approach is not practical.
- Their observation is noisy.
- Go to a location as close as to possible to that location also as close as possible to each other.



Example

- A camera sends an observation of the object.
- What is the value of this common observation?

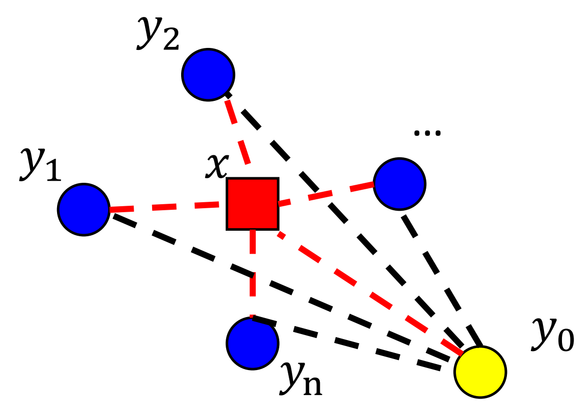


Literature Review

- **Static Teams**
  - Marschak (1955): Radner (1962): Marschak and Radner (1972).
- **Dynamic Teams**
  - Witsenhausen (1969,1971,1972…).
  - Ho and Chu (1972); Chu (1972): Partially nested information structures.
  - Krainak and Marcus (1982): Krainak et al. (1982): Risk sensitive teams.
  - Sandell and Athans (1974): Yoshikawa (1975)…: one-step delay information sharing.
  - …
  - Yuksel (2009): Stochastically nested information structures.
  - Nayyar et al. (2013): Partial history sharing.

Static team with common information

- x is the state of nature.
- n agents. Agent i observes y_i.
- A sensor observes y_0 and broadcasts it.
- $x, y_0, y_1, \ldots, y_n$ Jointly Gaussian.



- $u_i = g_i(y_0, y_i)$.
- The performance:

$$c(x, u_1, \ldots, u_n) = \sum_{i \in N} \sum_{j \in N} u_i^T R_{ij} u_j + 2 \sum_{i \in N} u_i^T P_i x$$

- Cost and covariance matrices are known to every one.

What is the best n tuple $(g_1, \ldots, g_n)$ that minimizes $E[c(x,u)]$?

Solutions-Theorem 1

We derived two optimal strategies:

- **Common information approach**

$$u_i = L_i(y_i - \hat{y}_i) + H_i \hat{x}_0$$

$\hat{x}_0 = E[x|y_0], \hat{y}_i = E[y_i|y_0]$

- **Hierarchical approach**

$$u_i = v_i + \tilde{u}_i$$

- A higher-level coordinator observes y_0 and sends global correction signals v_i to agent i.

$$v_i = G_i(y_0 - \bar{y}_0) + H_i \bar{x}$$

- n lower-level agents observe y_i and choose a local control action

$$\tilde{u}_i = L_i(y_i - \bar{y}_i)$$

- The optimal cost:

$$J^* = -\hat{\eta}^T \hat{\Gamma}^{-1} \hat{\eta} - \bar{x}^T P^T R^{-1} P \bar{x} - Tr(\Theta_0 \Sigma_{00}^{-1} \Theta_0^T P^T R^{-1} P)$$

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$\hat{x}_0 = E[x|y_0], \hat{y}_i = E[y_i|y_0]$

- Gains depend only on the conditional covariances.
- Conditional covariances do not depend on the realization of the common information (RVs are Gaussian).
- Therefore, gains can be computed offline.

$$v_i = G_i(y_0 - \bar{y}_0) + H_i \bar{x}$$

- n lower-level agents observe y_i and choose a local control action

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Optimal Strategy- Alternative derivation

- **Common information approach**

$$u_i = F_i(\breve{x}_i + \bar{x}) + H_i\hat{x}_0$$

$\breve{x}_i = E[x|y_i - \hat{y}_i]$.

- **Hierarchical Approach**

$$u_i = \tilde{u}_i + v_i$$

- A higher-level coordinator observes y_0 and sends global correction signals v_i to agent i.

$$\hat{G}_i\hat{x}_0 + \hat{H}_i\bar{x}$$

- n lower-level agents observe y_i and choose a local control action

$$\tilde{u}_i = \hat{F}_i\hat{x}_i$$

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Optimal Strategy- Alternative derivation

- **Common information approach**

$$u_i = F_i(\breve{x}_i + \bar{x}) + H_i\hat{x}_0$$

$\breve{x}_i = E[x|y_i - \hat{y}_i]$.

- **Hierarchical Approach**

$$u_i = \tilde{u}_i + v_i$$

- Gains depend only on the conditional covariances.
- Useful when observations dimension is more than the state's dimension.
- Easily generalized to dynamic case.

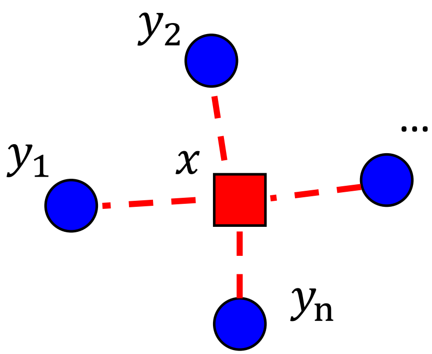
- n lower-level agents observe y_i and choose a local control action

$$\tilde{u}_i = \hat{F}_i\hat{x}_i$$

$\hat{x}_i = E[x|y_i]$.

Static team problem-Radner

- x is the state of nature.
- n agents. Agent i observes y_i.
- No common information.
- $x, y_1, \ldots, y_n$ Jointly Gaussian.



- $u_i = g_i(y_i)$.
- The performance:

$$c(x, u_1, \ldots, u_n) = \sum_{i \in N} \sum_{j \in N} u_i^T R_{ij} u_j + 2 \sum_{i \in N} u_i^T P_i x$$

What is the best n tuple $(g_1, \ldots, g_n)$ that minimizes $E[c(x, u)]$?

Optimal Strategy- Radner

The optimal decision rules are

$$u_i = L_i(y_i - \bar{y}_i) + H_i\bar{x}$$

- Gains depend only on the covariances.

Optimal value of the cost function

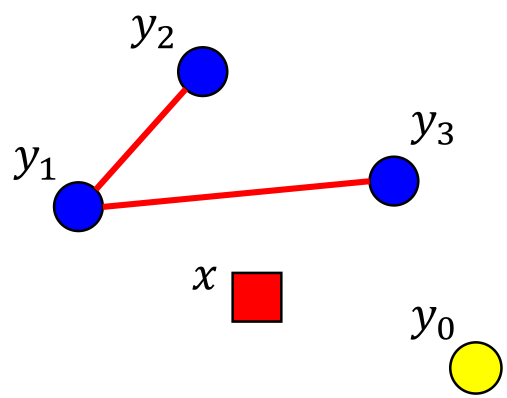
$$J^* = -\eta^T\Gamma^{-1}\eta - \bar{x}^T P^T R^{-1} P\bar{x}$$

We add the common information. The optimal control laws are

$$u_i = L_i\begin{bmatrix} y_0 - \bar{y}_0 \\ y_i - \bar{y}_i \end{bmatrix} + H_i\bar{x}$$

- $y_0 \in R^{d_y^0}, y = [y_1, \dots, y_n] \in R^{d_y}, u = [u_1, \dots, u_n] \in R^{d_u}$
- Then to find L_i we have to solve a systems of $d_u \times (d_y + nd_y^0)$ where in our results it is $d_u \times d_y$
- A system of higher order
- Requires more calculation than Theorem 1.

Example- Decentralized Estimation



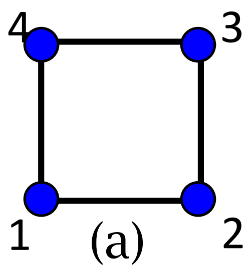
- $y_k = C_k x + w_k$
- $(x, w_0, \dots, w_n)$ are independent
- $x \sim (0, \Sigma_x)$ and $w_i \sim N(0, \sigma_i^2)$.
- Agent i generates an estimate of x.

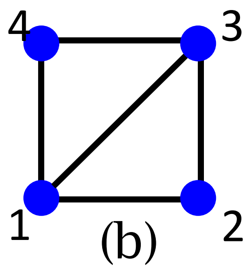
- The estimation error depends on how close the estimates are to x and how close are the estimates of neighboring agents.

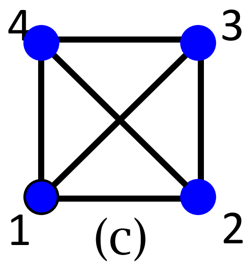
$$c(x, u_1, \dots, u_n) = \sum_{i \in N} (x - u_i)^T M_{ii} (x - u_i) + \sum_{i \in N} \sum_{j \in N, J > i} \left(u_i - u_j\right)^T M_{ij} (u_i - u_j)$$

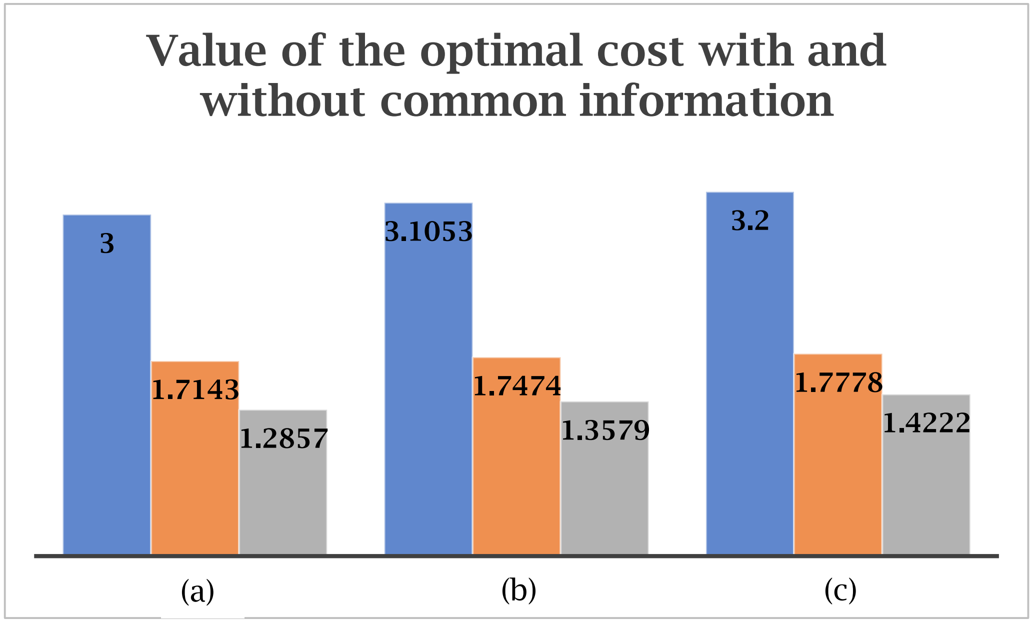
$$c(x, u) = (x - u_1)^2 + (x - u_2)^2 + (x - u_3)^2 + (u_1 - u_2)^2 + (u_1 - u_3)^2$$

Example- Decentralized Estimation



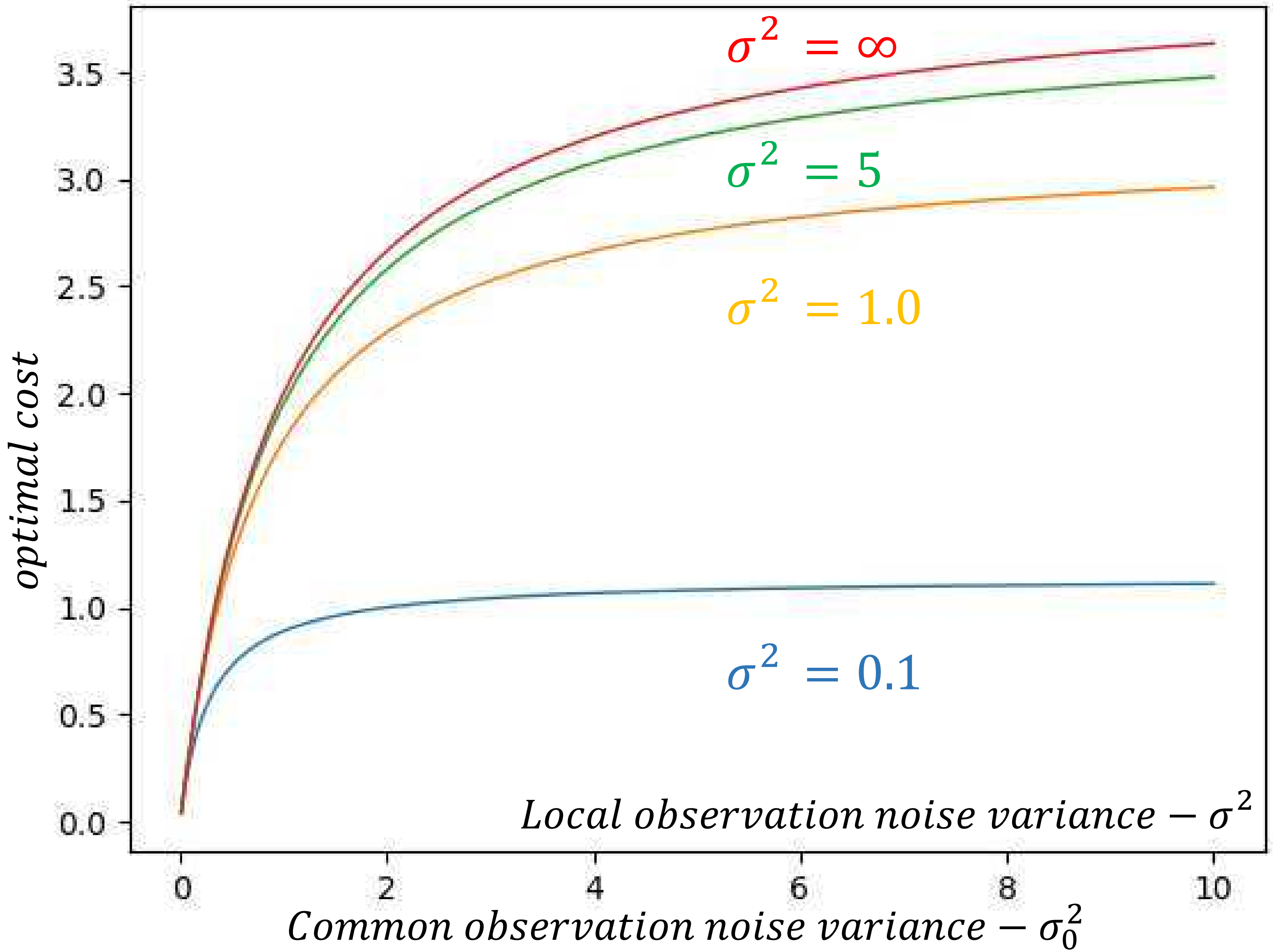




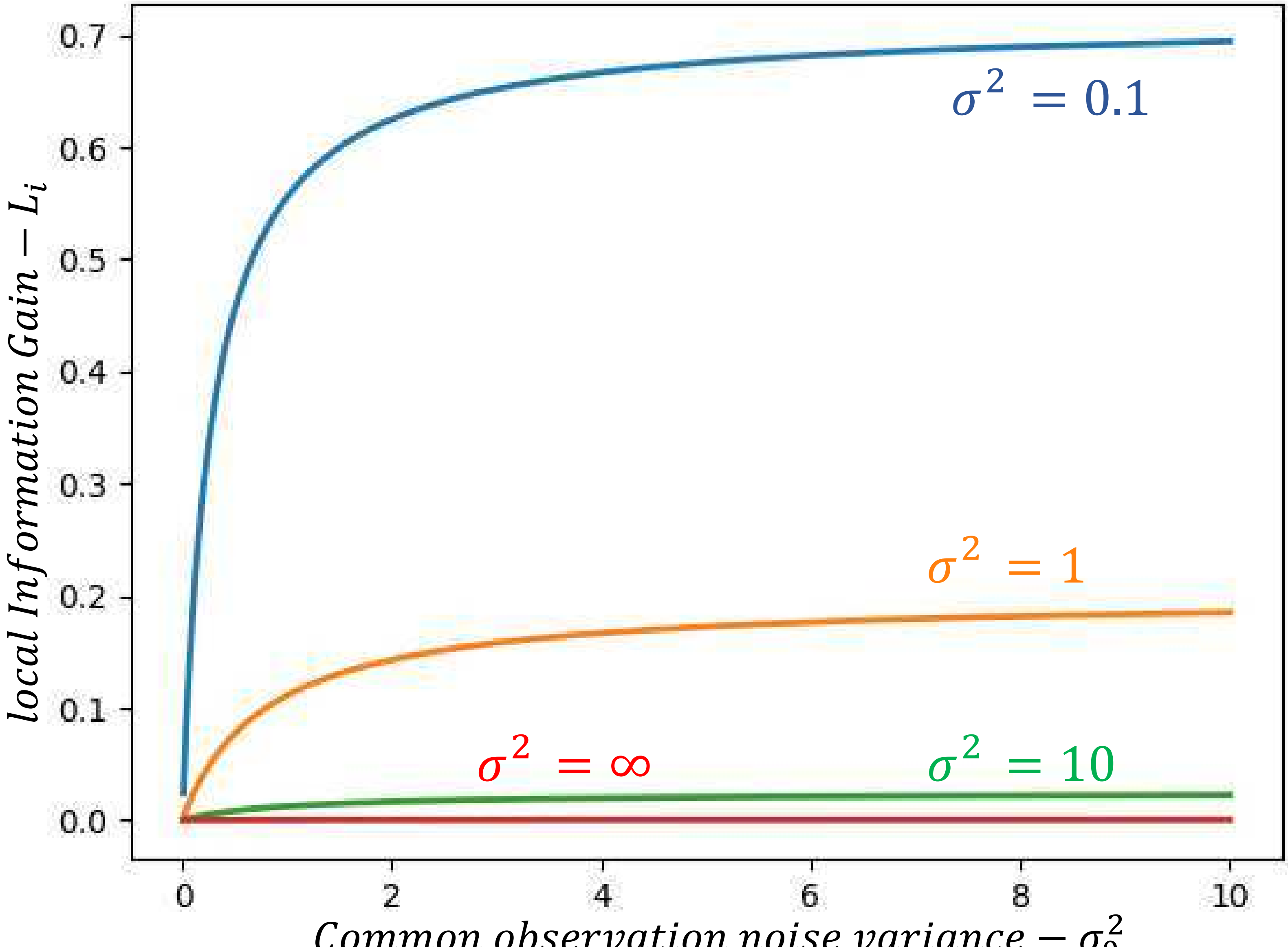


	u_i°	u_i^{*}
Case (a)	$u_i = \frac{1}{4} y_i$	$u_i = \frac{1}{7} y_i + \frac{3}{7} y_0$
Case(b)	$u_{\{1,3\}} = \frac{4}{19} y_i,$ $u_{\{2,4\}} = \frac{9}{38} y_i$	$u_{\{1,3\}} = \frac{11}{95} y_i + \frac{42}{95} y_0,$ $u_{\{2,4\}} = \frac{13}{95} y_i + \frac{41}{95} y_0$
Case (c)	$u_i = \frac{1}{5} y_i$	$u_i = \frac{1}{9} y_i + \frac{4}{9} y_0$

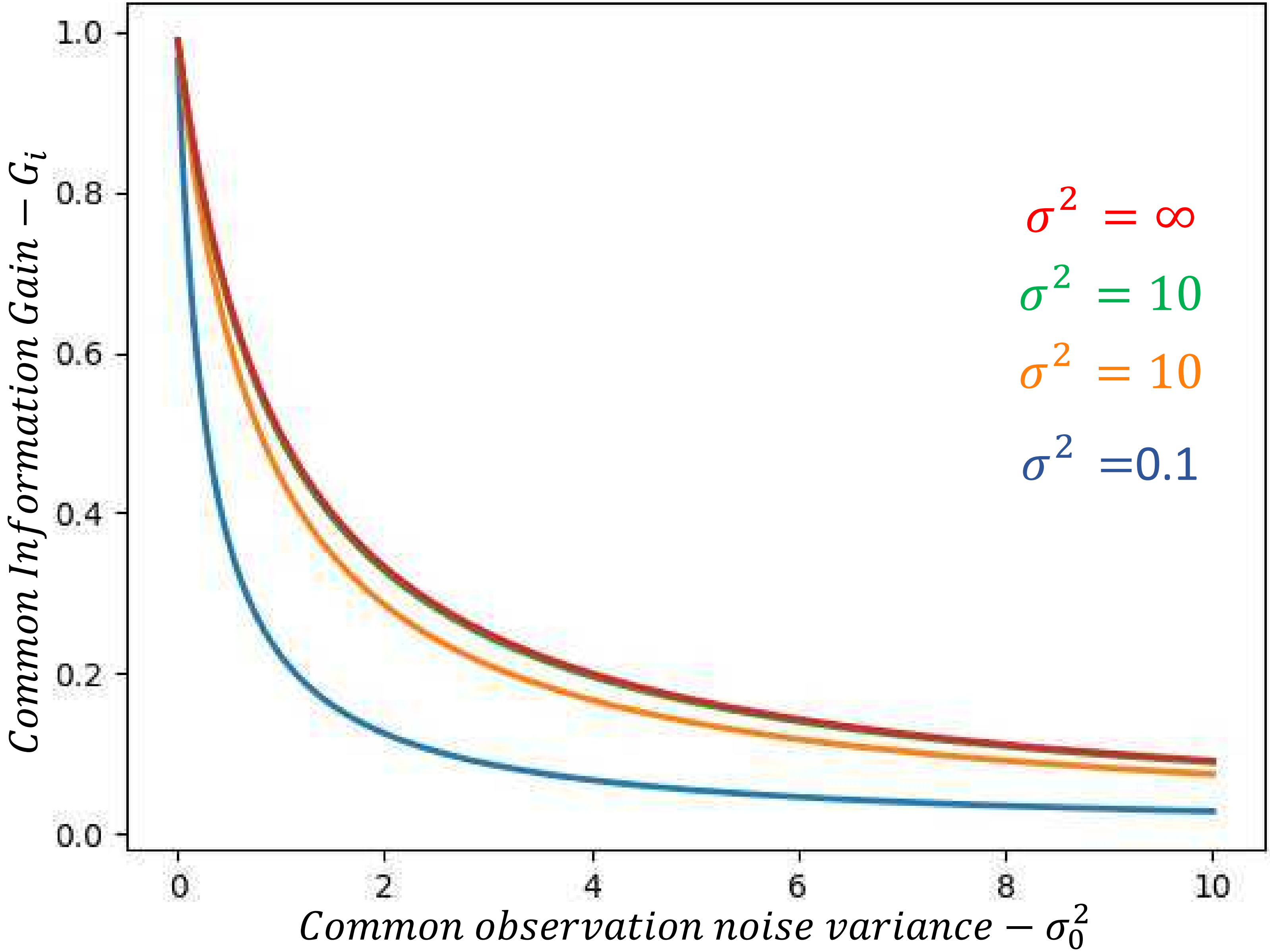
Example- Decentralized Estimation- $case(c)$



Example- Decentralized Estimation- $case(c)$



Example- Decentralized Estimation- $case(c)$



Conclusion/Future works

- Two methods to efficiently compute the optimal strategy and the optimal performance for teams with common information.
- Common information approach: solving a static team problem with the conditional means and covariances given the common information.
- Hierarchical approach: a local action as well as a global correction term.
- Hierarchical approach: when the common observation is high-dimensional (e.g., a video).
- The complexity is less than using the existing results for static teams.
- Optimal strategies with either the estimate of x or the observations.
- What is the solution to dynamic team problems using these approaches?

Thank you!