Identifying tractable decentralized control problems on the basis of information structure

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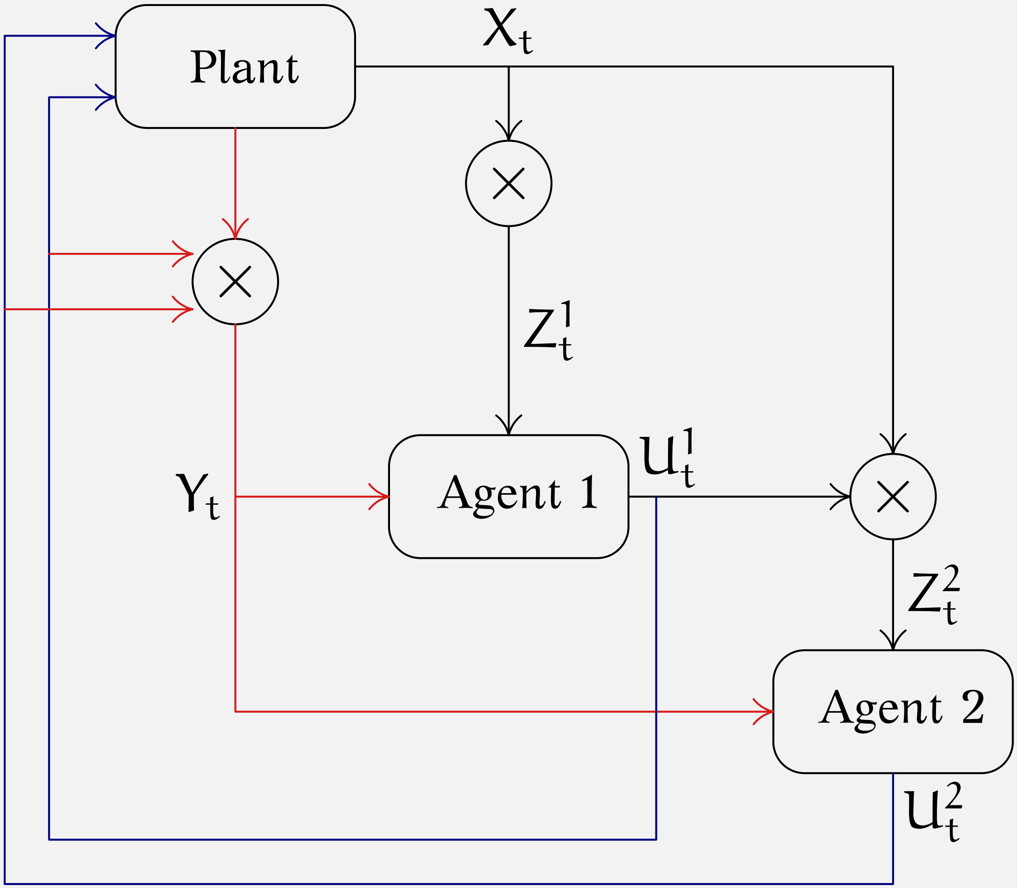
Optimal design of decentralized systems with non-classical information structures

- **Difficulties**: Conceptual and computational
- **Results of this paper**: Consider two general models of decentralized systems and obtain a sequential decomposition for their finite and infinite horizon cases.
- Our models encompass
  - Standard form (Witsenhausen, 1973)
  - k-step delay sharing pattern (Walrand and Varaiya, 1978)
  - Generic team model of Witsenhausen (1988)
- **Main idea**: viewed appropriately, these models are equivalent to POMDPs with functions as control actions
- Numerical solution can be obtained using existing techiques for POMDPs

Model A for two agents



Model A for two agents

- **Plant**: $X_{t+1} = f_t(X_t, U_t^1, U_t^2, W_t)$

- **Observations**
  - Common message:

    $$Y_t = c_t(X_t, U_{t-1}^1, U_{t-1}^2, Q_t)$$

  - Private message: $Z_t^1 = h_t^1(X_t, N_t^1)$

    $$Z_t^2 = h_t^2(X_t, U_t^1, N_t^2)$$

- **Agent** k
  - Control: $U_t^k = g_t^k(Y^t, Z_t^k, M_{t-1}^k)$
  - Memory update: $M_t^k = l_t^k(Y^t, Z_t^k, M_{t-1}^k)$

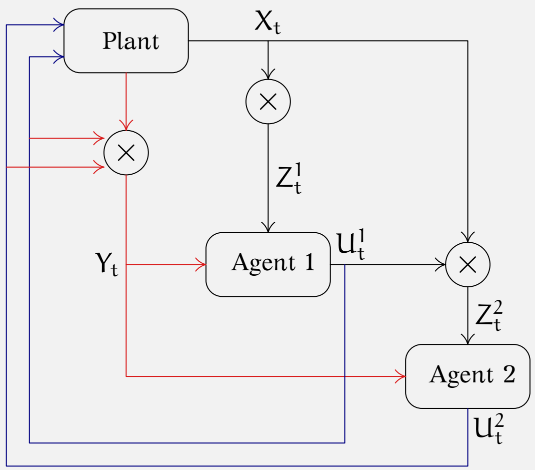
- **Design** $\equiv$ all control and memory update functions of both agents

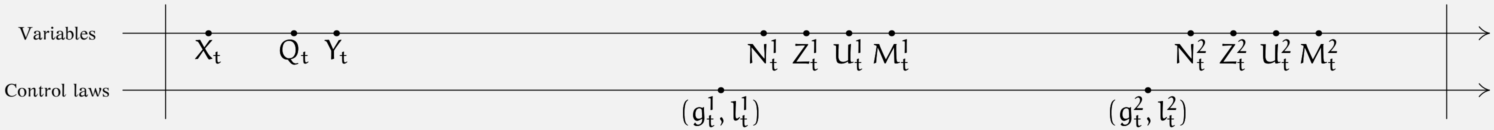
- **Cost at time** t: $\rho_t(X_t, U_t^1, U_t^2)$. **Cost of a design**: $E\left\{ \sum_{t=1}^{T} \rho_t(X_t, U_t^1, U_t^2) \,\middle|\, \text{Design} \right\}$

- **Objective**: Determine an optimal design

Model A for two agents

- Salient features
  - Non-classical information structures
  - Sequential system





Consider the model from the point of view of a fictitious common agent

Common Agent

Common agent observes all common messages

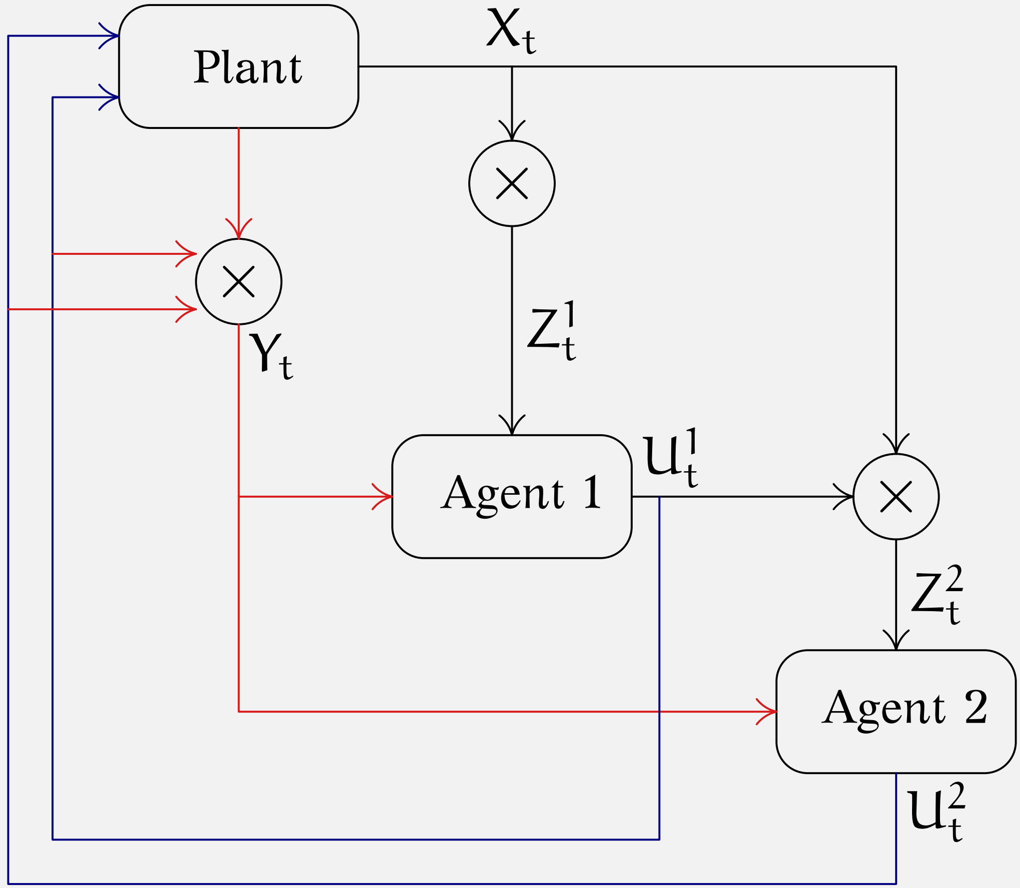
- Think of control and memory update functions in two steps

$$U_t^k = g_t^k(Y^t, Z_t^k, M_{t-1}^k)$$
$$= \hat{g}_t^k(Z_t^k, M_{t-1}^k), \quad \text{where } \hat{g}_t^k = \gamma_t^k(Y^t)$$

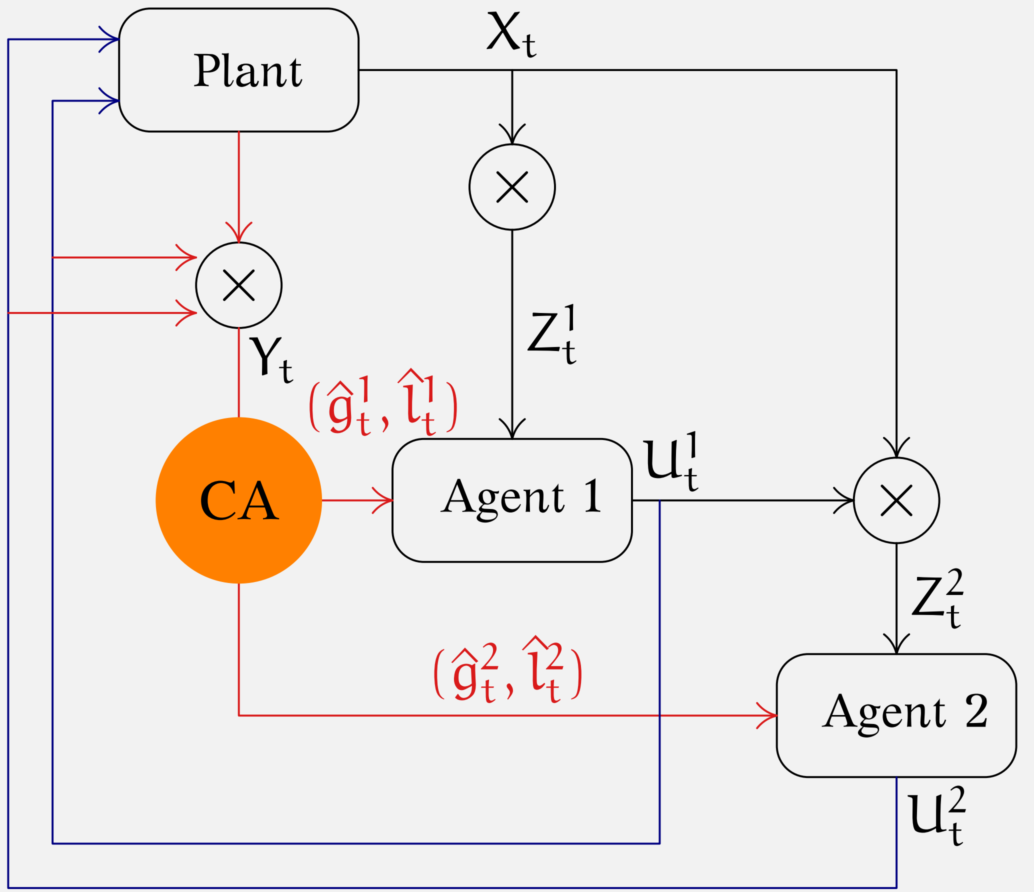
Similarly,

$$M_t^k = l_t^k(Y^t, Z_t^k, M_{t-1}^k)$$
$$= \hat{l}_t^k(Z_t^k, M_{t-1}^k), \quad \text{where } \hat{l}_t^k = \lambda_t^k(Y^t)$$

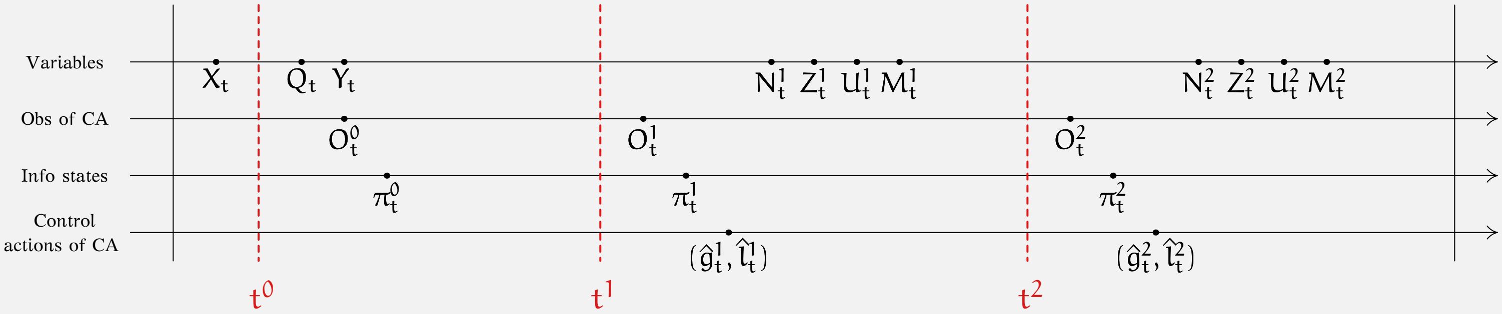
Common Agent's viewpoint



Common Agent's viewpoint



Common Agent's viewpoint



- Consider three time steps t^0, t^1, and t^2 in time interval t

$$S_t^0 = (X_t, M_{t-1}^1, M_{t-1}^2, U_{t-1}^1, U_{t-1}^2), \qquad O_t^0 = Y_t$$

$$S_t^1 = (X_t, M_{t-1}^1, M_{t-1}^2), \qquad O_t^1 = -$$

$$S_t^2 = (X_t, M_t^1, M_{t-1}^2, U_t^1), \qquad O_t^2 = -$$

- POMDP with: ▷ **State**: S_t^i, ▷ **Obs**: O_t^i, ▷ **Control actions**: $(\hat{g}_t^k, \hat{l}_t^k)$

From the common agent's viewpoint $\{S_t^0, S_t^1, S_t^2, t = 1, \ldots, T\}$ is a POMDP (partially observable Markov decision process)

Sequential decomposition

- Information states

$$\pi_t^0 = \Pr\left(S_t^0 \,\middle|\, Y^t, \hat{g}^{1,t-1}, \hat{l}^{1,t-1}, \hat{g}^{2,t-1}, \hat{l}^{t-1}\right)$$

$$\pi_t^1 = \Pr\left(S_t^1 \,\middle|\, Y^t, \hat{g}^{1,t-1}, \hat{l}^{1,t-1}, \hat{g}^{2,t-1}, \hat{l}^{t-1}\right)$$

$$\pi_t^2 = \Pr\left(S_t^2 \,\middle|\, Y^t, \hat{g}^{1,t}, \hat{l}^{1,t}, \hat{g}^{2,t-1}, \hat{l}^{t-1}\right)$$

- Optimality equations

$$V_{T+1}^0(\pi_{T+1}^0) \equiv 0,$$

for $t = 1, \ldots, T$

$$V_t^0(\pi_t^0) = E\left\{V_t^1(\pi_t^1) \,\middle|\, \pi_t^0\right\},$$

$$V_t^1(\pi_t^1) = \min_{\theta_t^1}\left\{ E\left\{V_t^2(\pi_t^2) \,\middle|\, \pi_t^1, \theta_t^1\right\} \right\},$$

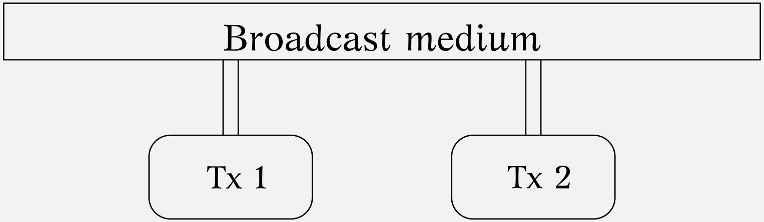
$$V_t^2(\pi_t^2) = \min_{\theta_t^2}\left\{ E\left\{\rho_t(X_t, U_t^1, U_t^2) + V_{t+1}^0(\pi_{t+1}^0) \,\middle|\, \pi_t^2, \theta_t^2\right\} \right\},$$

where $\theta_t^k = (\hat{g}_t^k, \hat{l}_t^k)$

Models considered in the paper

- Model A
  - n-agent version of what was presented here
- Model B
  - Model A with no common messages
- Also consider infinite horizon problems

Example – multiaccess broadcast



- MAB Channel
  - Single user transmits $\Longrightarrow$ successful transmission
  - Both users transmit $\Longrightarrow$ packet collision
- Transmitters
  - Queues with buffer size 1
  - Packet held in queue until successful transmission
  - Packet arrival is independent Bernoulli process

Example – multiaccess broadcast

- Channel feedback

  Both transmitters know if there was no transmission, successful transmission, or a collision

- Policy of transmitters

  If packet is available, decide whether or not to transmit based on all past channel feedback

- Objective: Maximize throughput
  - Avoid collisions
  - Avoid idle

History of multiaccess broadcast

- Hluchyj and Gallager,

  "Multiaccess of a slotted channel by finitely many users", NTC 81.

  - Considered symmetric arrival rates
  - Restricted attention to "window protocols"

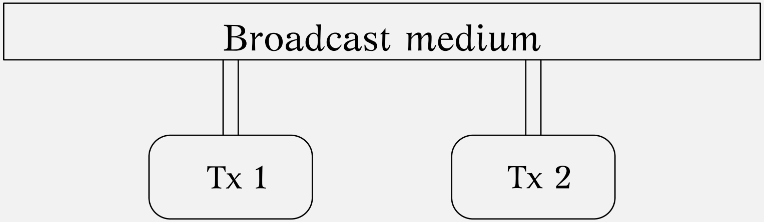
- Ooi and Wornell,

  "Decentralized control of multiple access broadcast channels", CDC 96.

  - Considered a relaxation of the problem
  - Numerically find optimal performance of the relaxed problem
  - Hluchyj and Gallager's scheme meets this upper bound

- AI Literature
  - Consider the case of asymmetric arrival rates
  - Approximate heuristic solutions for small horizons

Multi-access broadcast is equivalent to Model A



Tx 1	$\equiv$	Agent 1
Tx 2	$\equiv$	Agent 2
Channel feedback	$\equiv$	Common message
Number of packets in each buffer	$\equiv$	Private messages

- **Information state**: $\pi_t = \Pr\left(Z_t^1, Z_t^2 \,\middle|\, \text{feedback}\right), \quad Z_t^k = \{0, 1\}$
- **Action Space**: $\hat{g}_t^k : \{0, 1\} \to \{\text{Tx, Don't Tx}\}$

Equivalent to a POMDP with finite state and action spaces

Tractability

- Finite horizon problem
  - All system variables are **finite valued**
- Infinite horizon
  - All system variables take values in a **time-invariant** space
  - The system is **time-homogeneous**

Conclusions

- Sequential decomposition of two general models of decentralized systems
- Equivalent to POMDPs (sometimes to POMDPs with finite state and action spaces)
- Harder to solve than POMDPs due to expansion of state and action spaces.

Thank you