# Compound channel with feedback: Opportunistic capacity and error exponents

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# Compound channel

- Channel model

$$\mathbb{P}(Y_n \mid X^n, Y^{n-1}) = Q_\circ(Y_n|X_n)$$

$Q_\circ \in \mathbb{Q}$, $\mathbb{Q}$ known to encoder and decoder

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$$C(\mathbb{Q}) = \max_{P \in \Delta(\mathbb{X})} \inf_{Q \in \mathbb{Q}} I(P, Q)$$

- Capacity **with feedback**

$$C_F(\mathbb{Q}) = \inf_{Q \in \mathbb{Q}} \max_{P \in \Delta(\mathbb{X})} I(P, Q)$$

Feedback capacity is defined pessimistically

# Outline

- Variable length coding scheme
  - Achievable rate and opportunistic capacity
  - Probability of error and error exponents
- Literature Overview
  - Variable length communication over DMC
  - Variable length communication over compound channel
- Main Result
  - Lower bound on error exponent region
  - Achievable coding scheme
- Example

# Variable length coding

Assume $\mathbb{Q} = \{Q_1, ..., Q_L\}$. Variable length coding scheme is a tuple $(\mathbf{M}, \mathbf{f}, \mathbf{g}, \tau)$

- Compound message: $\mathbf{M} = (M_1, ..., M_L)$. Let $\mathbb{M} = \prod_{\ell=1}^{L} \{1, ..., M_\ell\}$.
- Encoding strategy: $\mathbf{f} = (f_1, f_2, ...)$

$$f_t = \mathbb{M} \times \mathbb{Y}^{t-1} \mapsto \mathbb{X}$$

- Decoding strategy: $\mathbf{g} = (g_1, g_2, ...)$

$$g_t : \mathbb{Y}^t \mapsto \bigcup_{\ell=1}^{L} \{(\ell, 1), (\ell, 2), ..., (\ell, M_\ell)\}$$

- Stopping time $\tau$ with respect to the channel output $Y^t$

# Operation of the scheme

- Compound message $\mathbf{W} = (W_1, \ldots, W_L)$
- $W_\ell$ is uniformly distributed in $\{1, \ldots, M_\ell\}$
- Encoder

$$X_1 = f_1(\mathbf{W}), \quad X_2 = f_2(\mathbf{W}, Y_1), \quad \cdots$$

- Decoder:

$$(\hat{L}, \hat{W}) = g_\tau(Y_1, \ldots, Y_\tau)$$

# Performance metrics

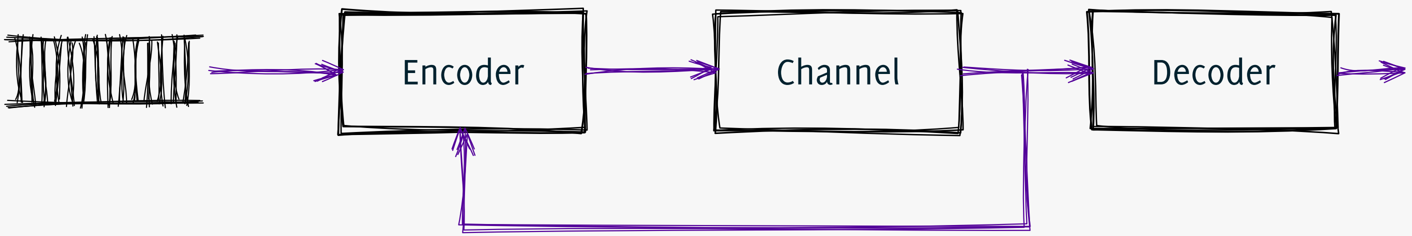
- Probability of error $\mathbf{P} = (P_1, \ldots, P_L)$

$$P_\ell = \mathbb{P}_\ell(\hat{W} \neq W_{\hat{L}})$$

- Rate: $\mathbf{R} = (R_1, \ldots, R_L)$

$$R_\ell = \frac{\mathbb{E}_\ell[\log M_{\hat{L}}]}{\mathbb{E}_\ell[\tau]}$$

# Operational interpretation




- Variable length communication using $(\mathbf{M}, \mathbf{f}, \mathbf{g}, \tau)$
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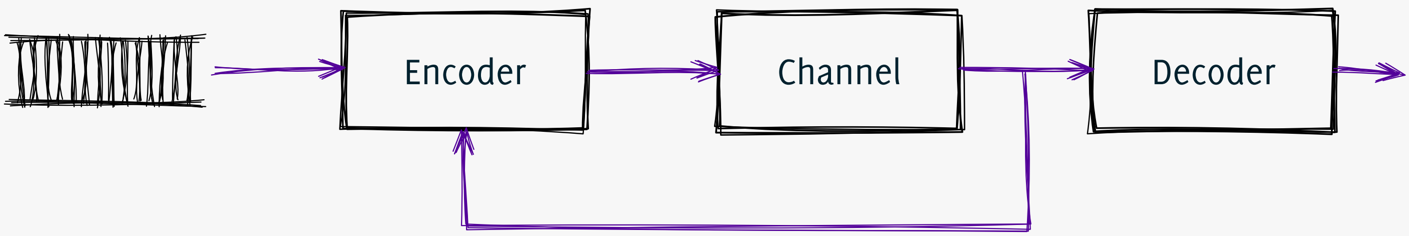


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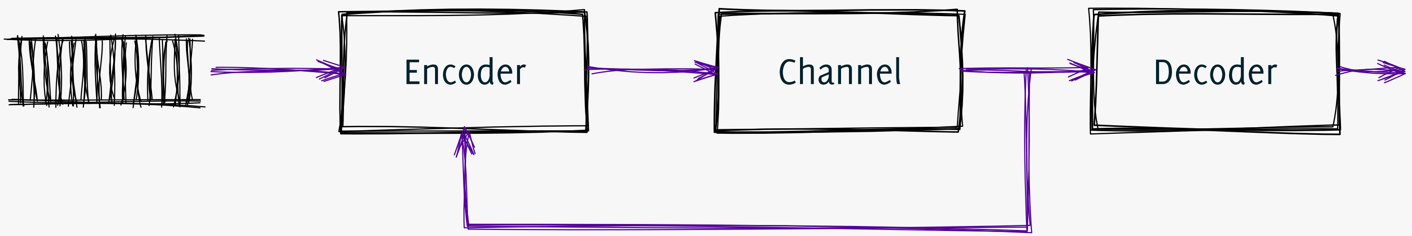


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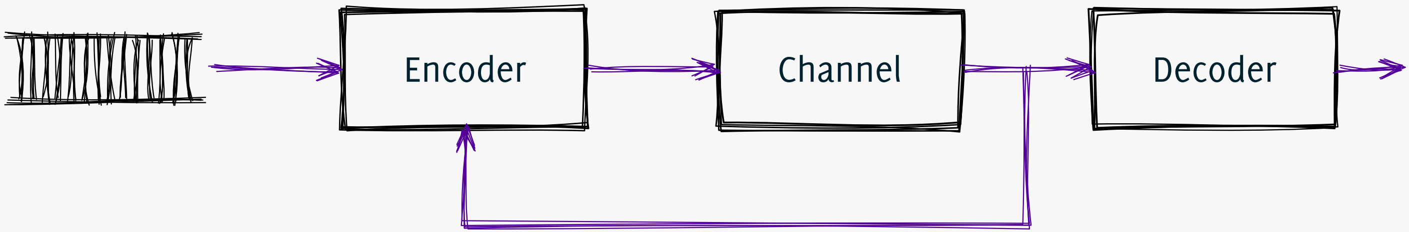
  - Let $M_{\text{max}} = \max\{M_1, ..., M_L\}$ and $M_{\text{min}} = \min\{M_1, ..., M_L\}$
- Encoding
  - Transmitter picks $\log_2 M_{\text{max}}$ bits from the bit-stream.
  - $W_\ell$ is the decimal expansion of the first $\log_2 M_\ell$ of these bits.

# Operational interpretation




## Decoding

- At stopping time $\tau$, the receiver passes $(\hat{W}, \hat{L})$ to a higher layer application.
- The transmitter removes $\log_2 M_{\hat{L}}$ bits from the $\log_2 M_{\max}$ initially chosen bits and returns the remaining bits to the bit-stream.

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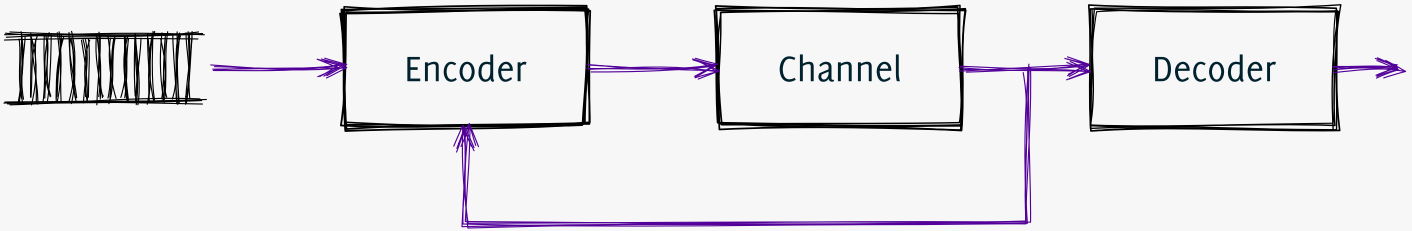


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Advantage of being opportunistic: $\log_2 M_{\hat{L}} - \log_2 M_{\min}$

# Opportunistic capacity

## Achievable Rate

Rate $\mathbf{R} = (R_1, \ldots, R_L)$ is **achievable** if $\exists$ sequence of coding schemes such that for $\varepsilon > 0$ and sufficiently large $n$, and for all $\ell = 1, \ldots, L$,

1. $\lim_{n\to\infty} \mathbb{E}_\ell[\tau^{(n)}] = \infty$
2. $P_\ell^{(n)} < \varepsilon$ and $R_\ell^{(n)} \geq R_\ell - \varepsilon$

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The union of all achievable rates is called the **opportunistic capacity region** $\mathbb{C}_F(\mathbb{Q})$

# Error Exponents

## Error exponent

Given a sequence of coding scheme that achieve a rate vector $\mathbf{R}$, the error exponent $\mathbf{E} = (E_1, \ldots, E_L)$ is given by

$$E_\ell = \lim_{n\to\infty} -\frac{\log P_\ell^{(n)}}{\mathbb{E}_\ell[\tau^{(n)}]}$$

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For a particular rate $\mathbf{R}$, the union of all possible error exponents is called the **error exponent region** $\mathbb{E}(\mathbf{R})$.

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# Variable length communication over DMC

Special case of a compound channel when $|\mathbb{Q}| = 1$.

- Burnashev-76, "Data transmission over a discrete channel with feedback: Random transmission time"

**Burnashev** exponent

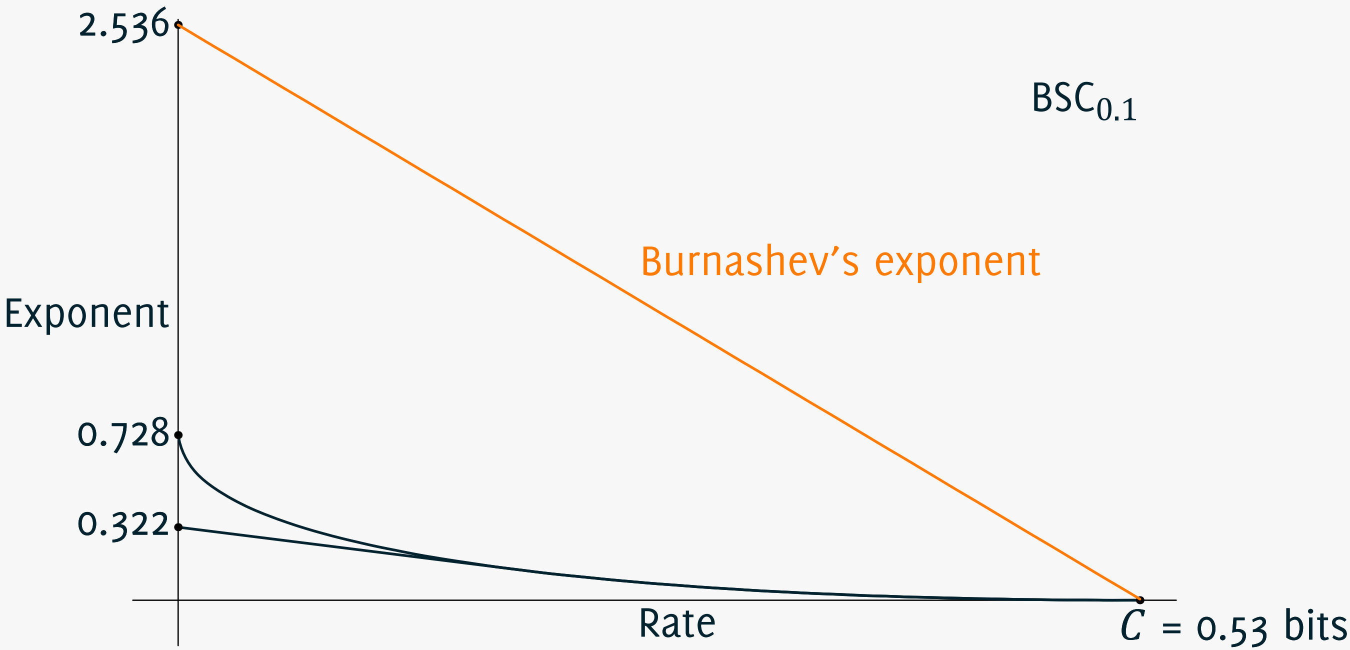
$$E(R, Q) = B_Q(1 - \gamma)$$

where $\gamma = R/C$.

# Variable length communication over DMC

- Achievability scheme
  - Yamamoto-Itoh-79, "Asymptotic performance of a modified Schalkwijk-Barron scheme with noiseless feedback".
  - **Message mode**: Fixed length code at rate $C - \varepsilon$ and length $\gamma n$
  - **Control mode**: Send ACCEPT or REJECT for length $(1 - \gamma)n$
  - Repeat until ACCEPT is received

# Advantage of variable length comm



# Variable length comm over compound channel

- Tchamkerten-Telatar-06, “Variable length coding over unknown channel”

> Can we achieve Burnashev exponent
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- Tchamkerten-Telatar-06, "Variable length coding over unknown channel"

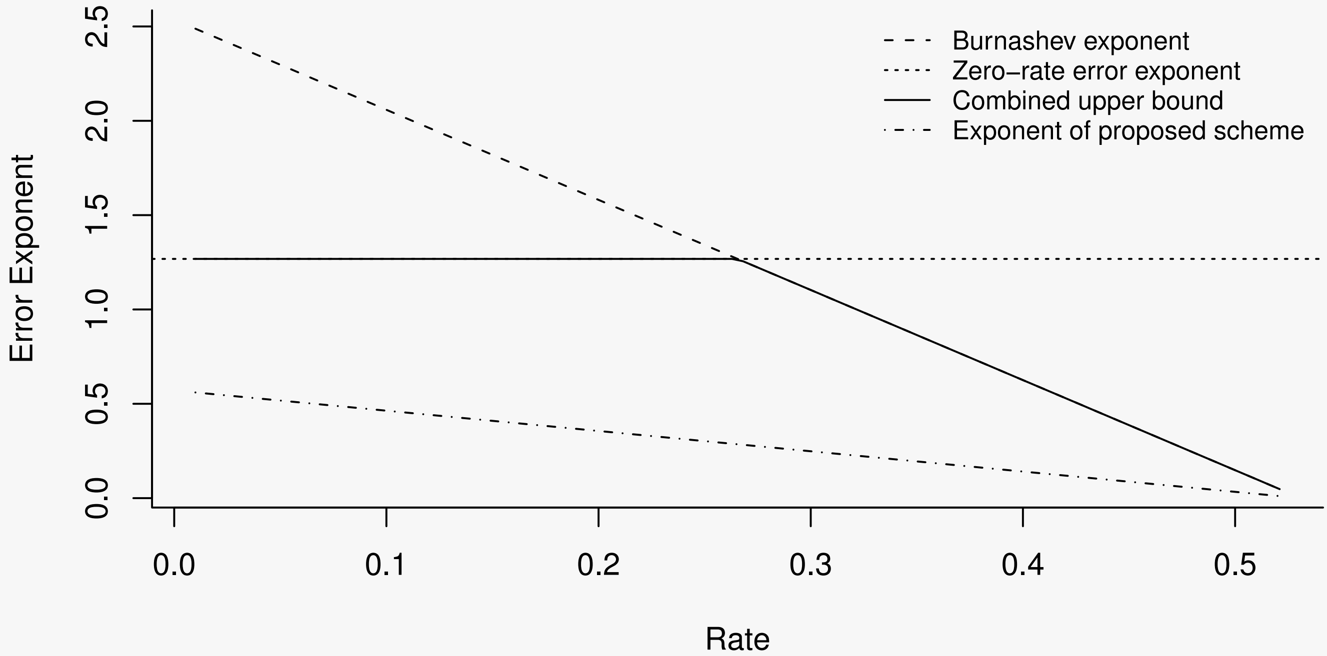
Can we achieve Burnashev exponent
even if we do not know the channel?

- Negative result

Restricted attention to $R_\ell / C_\ell = constant$

- Under some restricted conditions, yes.
- In general, no.

# Counterexample: $\{\mathrm{BSC}_p, \mathrm{BSC}_{1-p}\}$



# Questions

- What are the error exponents when conditions of Tchamkerten-Telatar-06 are not met?
- Which coding schemes achieve the best exponent?
- What about rates when $R_\ell / C_\ell$ is not a constant?

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# Main Result

- Opportunistic Capacity

$$\mathbb{C}_F(\mathbb{Q}) = \{(R_1, \ldots, R_L) : 0 \leq R_\ell < C_\ell, \ell = 1, \ldots, L\}$$

where $C_\ell$ is the capacity of DMC $Q_\ell$.

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## Error Exponent Region

- Let $T_\ell^c$ be the exponent of the channel estimation error when the channel is $Q_\ell$. For any channel estimation scheme, $(T_1^c, \dots, T_L^c) \in \mathbb{T}^*$.

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- **Error Exponent Region**
  - Let $T_\ell^c$ be the exponent of the channel estimation error when the channel is $Q_\ell$. For any channel estimation scheme, $(T_1^c, \ldots, T_L^c) \in \mathbb{T}^*$.
  - At rate $\mathbf{R} = (R_1, \ldots, R_L)$, the error exponent is

$$E_\ell \geq \frac{T_\ell^c}{T_\ell^c + B_{Q_\ell}} B_{Q_\ell} \left(1 - \frac{R_\ell}{C_\ell}\right)$$

where $B_{Q_\ell} = \max_{x_A, x_R \in \mathbb{X}} D(Q_\ell(\cdot|x_A), Q_\ell(\cdot|x_R))$

# The achievable scheme


Communicate in variable number of epochs. Each epoch is variable length and consists of four phases

- Training phase of length $\beta_1(n)n$. Generate channel estimate $\hat{L}_m$
- Message phase of length $\beta_2(\hat{L}^m, n)n$. Assume that the channel is $\hat{L}_m$.
- Re-training phase of length $\beta_3(n)n$. Generate channel estimate $\hat{L}_c$.
- Control phase of length $\beta_4(\hat{L}_c, n)n$. Transmit ACCEPT or REJECT assuming that the channel is $\hat{L}_c$.

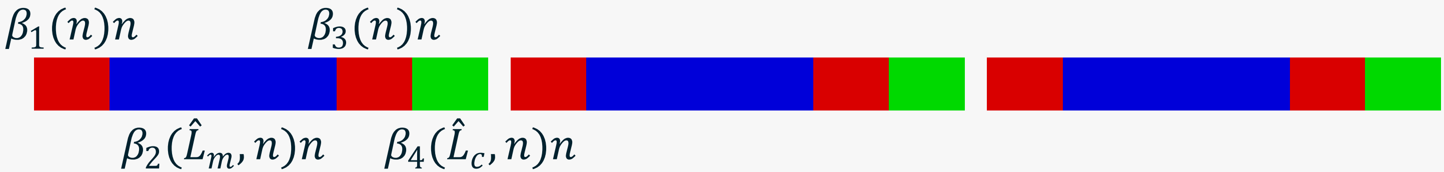
# Proof Outline: Number of epochs


- Number of epochs $K(n)$

$$\mathbb{P}_\ell(K(n) = k) = p_\ell(n)(1 - p_\ell(n))^{k-1}, \qquad \lim_{n\to\infty} p_\ell(n) = 1$$

**Number of epochs $\approx 1$**

# Proof Outline: Rate of transmission

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$$\lim_{n\to\infty} \frac{\mathbb{E}_\ell[\#\ \text{messages}]}{\mathbb{E}_\ell[\#\ \text{epochs}]\mathbb{E}_\ell[\text{epoch length}]}$$

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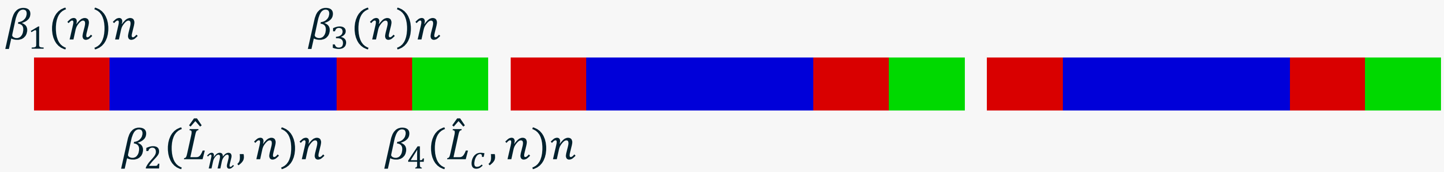


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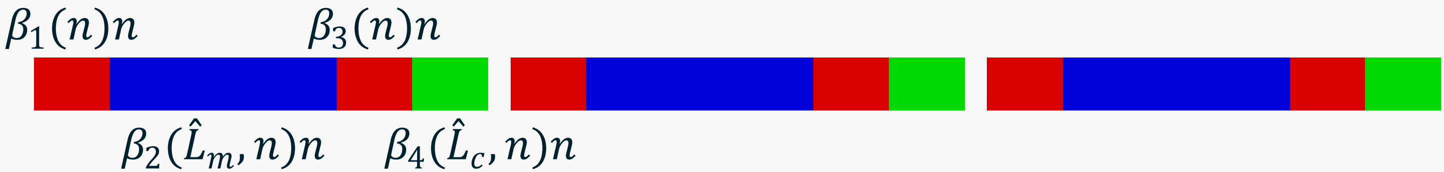


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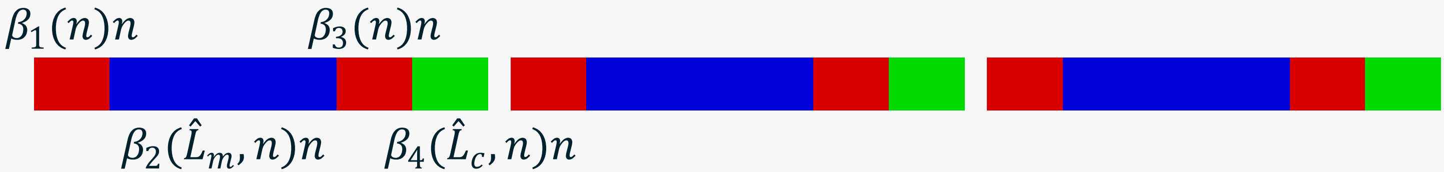


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- Error exponent

$$\lim_{n\to\infty} \frac{-\log P_\ell}{\mathbb{E}_\ell[\#\text{ epochs}]\mathbb{E}_\ell[\text{epoch length}]} \geq \frac{T_\ell^c \cdot B_{Q_\ell}}{T_\ell^c + B_{Q_\ell}}(1-\gamma_\ell)$$

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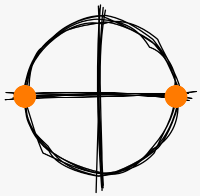

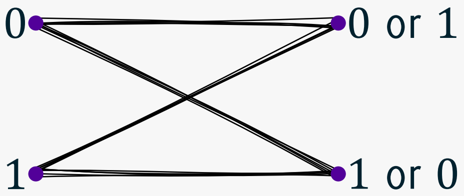


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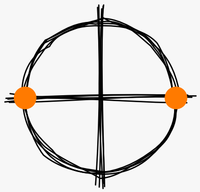

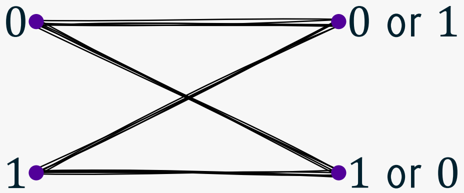


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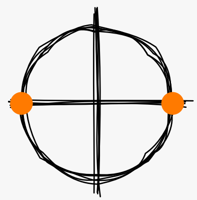

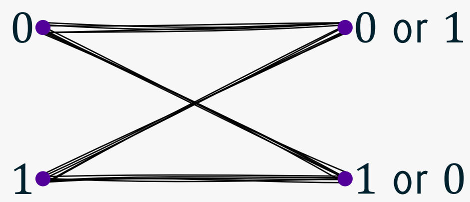


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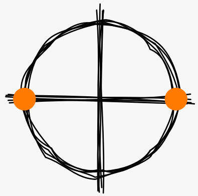
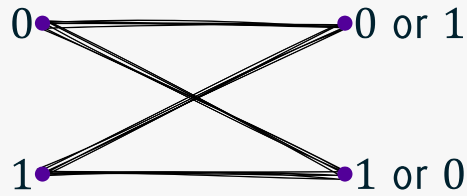
# An Example






- $\mathbb{Q} = \{\mathrm{BSC}_p, \mathrm{BSC}_{1-p}\}$, $p$ known at encoder and decoder
- **Capacity:** $C_p = C_{1-p} = 1 - h(p)$
- **Slope of Burnashev exp:** $B_p = B_{1-p} = D(p \| 1-p)$
- **Channel estimation rule:** Transmit the all zero sequence as the training sequence. Estimate $\mathrm{BSC}_p$ if frequency of ones is less than $q$; else estimate $\mathrm{BSC}_{1-p}$.

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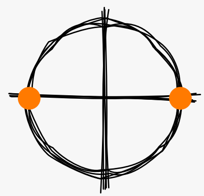

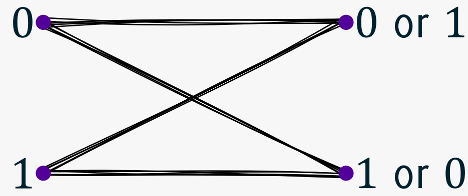


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- **Exponent of training error**:

$$T_p = D(p\|q) \qquad T_{1-p} = D(1-p\|q)$$

# Performance evaluation

Communication at rate $\mathbf{R} = (R_p, R_{1-p})$. Let $\gamma = R/C$.

- Error exponents

$$E_p \geq \frac{D(q\|p) \cdot D(p\|1-p)}{D(q\|p) + D(p\|1-p)}(1-\gamma_p)$$

$$E_{1-p} \geq \frac{D(q\|1-p) \cdot D(p\|1-p)}{D(q\|1-p) + D(p\|1-p)}(1-\gamma_{1-p})$$

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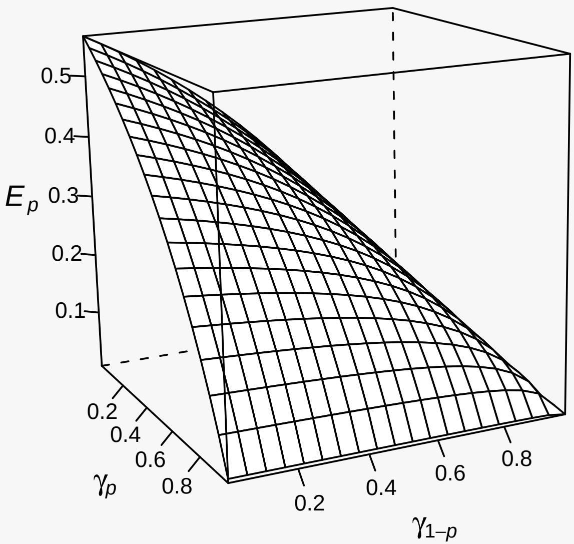
## Optimal threshold $q$

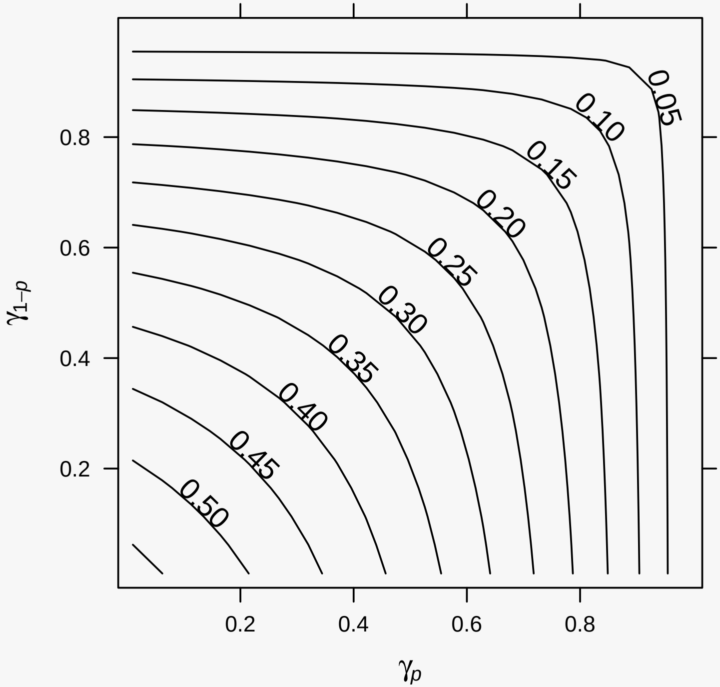
Choose $q$ such that $E_p = E_{1-p}$: solve for $q$ in

$$\varphi(q,p) = \frac{(1-\gamma_p)}{(1-\gamma_{1-p})}$$

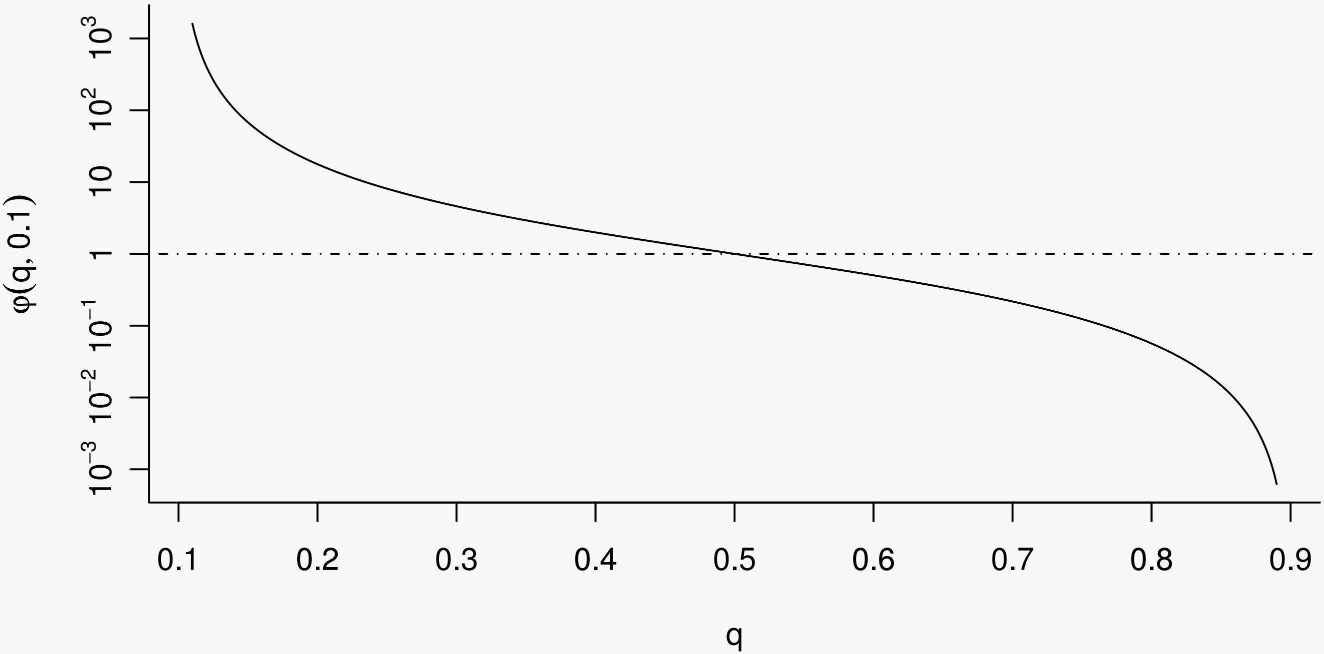
where $\varphi(q,p)$ is appropriately defined

# Error Exponents



# Threshold for channel estimation



# Threshold for channel estimation

| $\gamma_p$ | $\gamma_{1-p}$ | $q$ | $E_p = E_{1-p}$ |
|---|---|---|---|
| 0.5 | 0.1 | 0.5861 | 0.3666 |
| 0.5 | 0.2 | 0.5695 | 0.3511 |
| 0.5 | 0.3 | 0.5501 | 0.3329 |
| 0.5 | 0.4 | 0.5273 | 0.3114 |
| 0.5 | 0.5 | 0.5000 | 0.2855 |
| 0.5 | 0.6 | 0.4666 | 0.2537 |
| 0.5 | 0.7 | 0.4247 | 0.2139 |
| 0.5 | 0.8 | 0.3698 | 0.1628 |
| 0.5 | 0.9 | 0.2918 | 0.0952 |

# Conclusion

## Contributions

- Defining opportunistic capacity and corresponding error exponent regions for compound channels with feedback.
- A simple and easy to implement coding scheme whose error exponents are within a multiplicative factor of the best possible error exponents
- In the presence of feedback, training based schemes can lead to reasonable performance

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## Future directions

- Channels defined over continuous families and continuous alphabets
- Upper bound on error exponents

Thank You